Time value of money formulas

Prepared by Pamela Peterson Drake

1. Time value of a lump-sum

A. Discrete compounding

$$FV = PV\,(1+i)^n$$

where FV is the future value,
PV is the present value
i is the rate of interest, and
n is the number of compounding periods

$$PV = \frac{1}{(1+i)^n}$$

$$i = \left(\sqrt{FV/PV}\right) - 1 \qquad n = \frac{\ln FV - \ln PV}{\ln(1+i)}$$

B. Continuous compounding

$$FV = PV\, e^{APR\; x}$$

where x is the number of years
APR is the annual percentage rate
e Euler's e

$$PV = \frac{FV}{e^{APR\; x}}$$

2. Time value of annuities

A. Ordinary annuity

$$FV = \sum_{t=1}^{N} CF(1+i)^{N-t} = CF\sum_{t=1}^{N}(1+i)^{N-t}$$

$$PV = \sum_{t=1}^{N}\frac{CF}{(1+i)^t} = CF\sum_{t=1}^{N}\frac{1}{(1+i)^t} = CF\left(1 - \frac{\left(1+i\right)^N}{i}\right)$$

where $\sum_{t=1}^{N}\frac{1}{(1+i)^t}$ is the annuity discount factor; and

$$\sum_{t=1}^{N}(1+i)^{N-t}$$ is the annuity compound factor.

B. Annuity due

$$FV = CF(1 + i)^1 + CF(1 + i)^2 + \ldots + CF(1 + i)^{N+1} = CF\left(\sum_{t=0}^{N}(1+i)^{t+1}\right)$$

$$PV = \sum_{t=1}^{N}\frac{CF}{(1+i)^{t-1}} = CF\sum_{t=1}^{N}\frac{1}{(1+i)^{t-1}}$$

C. Perpetuity

$$PV = \frac{CF}{i}$$

3. Determining annual interest rates

A. Discrete compounding

$APR = i \times n$

$EAR = (1 + i)^n \; 1 = (1 + {}^{APR}/_{n})^n - 1$

B. Continuous compounding

$EAR = e^{APR} - 1$