



Duration Problem Set

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Formulas

$$\text{Macauley duration} = \sum_{t=1}^{2N} \frac{t \text{ CF}_t}{\left(1 + \frac{\text{YTM}}{2}\right)^t}$$

$$\text{Modified duration} = \frac{\text{Macauley duration}}{\left(1 + \frac{\text{YTM}}{2}\right)}$$

$$\text{Effective duration} = \frac{P_{1\% \text{ decrease in yield}} - P_{1\% \text{ increase in yield}}}{2 (\text{initial price of security}) 0.01}$$

Problems

Calculate **modified duration** and **effective duration** for each of the following bonds:

Bond	Description
A	A coupon rate of 6%, maturity of ten years, and currently priced to yield 5%
B	A coupon rate of 6.5%, a maturity of five years, and currently priced to yield 4.3%
C	A coupon rate of 4.5%, a maturity of 10 years, and priced to yield 6.5%

Solutions

Bond	Macauley duration	Modified duration	Effective duration
A	7.7618	7.5725	7.5845
B	4.3833	4.2911	4.2932
C	4.5071	4.3653	4.3674

Example, Bond A

Period	Cash flow	Present value of cash flow	Present value scaled by price	Time weight	Time-weighted cash flow
1	30	29.26829	0.0272	0.5	0.0136
2	30	28.55443	0.0265	1.0	0.0265
3	30	27.85798	0.0258	1.5	0.0388
4	30	27.17852	0.0252	2.0	0.0504
5	30	26.51563	0.0246	2.5	0.0615
6	30	25.86891	0.0240	3.0	0.0720
7	30	25.23796	0.0234	3.5	0.0819
8	30	24.6224	0.0228	4.0	0.0914
9	30	24.02185	0.0223	4.5	0.1003
10	30	23.43595	0.0217	5.0	0.1087
11	30	22.86434	0.0212	5.5	0.1167
12	30	22.30668	0.0207	6.0	0.1242
13	30	21.76261	0.0202	6.5	0.1312
14	30	21.23182	0.0197	7.0	0.1379
15	30	20.71397	0.0192	7.5	0.1441
16	30	20.20875	0.0187	8.0	0.1500
17	30	19.71585	0.0183	8.5	0.1555
18	30	19.23498	0.0178	9.0	0.1606
19	30	18.76583	0.0174	9.5	0.1654
20	1030	628.5791	0.5831	10.0	<u>5.8313</u>
				Macauley=	7.7618

$$\text{Modified} = 7.7618 / 1.025 = \mathbf{7.5725}$$

Effective duration:

Value of bond at current YTM	\$1,077.95
Value of bond if YTM+1%	\$1,000.00
Value of bond if YTM-1%	\$1,163.51

$$\text{Effective duration} = (\$1,163.51 - 1,000) / (2 * 1,077.95 * 0.01) = \mathbf{7.5845}$$