



Module 9

Investing in bonds

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1. Overview

Long-term debt securities, such as notes and bonds, are legally binding commitments by the borrower/issuer to repay the principal amount when promised. Notes and bonds may also require the borrower to pay interest periodically, typically semi-annually or annually, and generally stated as a percentage of the face value of the bond or note. A bond's **term to maturity** is the number of years over which the issuer has promised the obligation's cash flows. The term to maturity is important in bond analysis because it determines the bond's cash flows. Further, the yield is related to the bond's maturity because of the yield-curve effect and, hence, a bond's price volatility is affected by the term to maturity.

The principal value is the amount that the issuer (i.e., borrower) agrees to repay at the maturity date. The principal amount of the loan is referred to as the **par value**, the **principal**, the **maturity value**, the **redemption value**, or the **face value**.

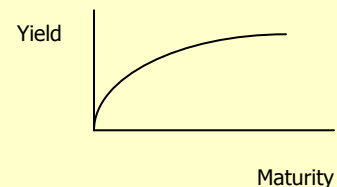
Bonds may be coupon bonds or zero-coupon bonds. In the case of a **coupon bond**, the issuer pays interest periodically, as a percentage of the bond's face value. We refer to the interest payments as **coupon payments** or **coupons** and the percentage rate as the **coupon rate**. If these coupons are a constant amount, paid at regular intervals, we refer to the security paying them as having a **straight coupon**. A debt security that does not have a promise to pay interest we refer to as a **zero-coupon** note or bond.

Though most corporate bonds are straight coupon bonds, the issuer may design an interest payment scheme for a bond that deviates from the semi-annual coupon payments. Variations in interest payments include:

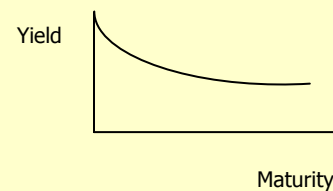
- **Deferred interest:** interest payments begin at some specified time in the future.
- **Step-up interest:** the coupon rate is low at the beginning of the life of the bond, but increases at a specified later point in time to a specified rate.
- **Payment-in-kind interest (PIK):** the investor has a choice of receiving cash or a similar bond in lieu of interest.
- **Reset interest:** the coupon rate is revised periodically as market interest rates change to force the price of the bond to a predetermined level.

THE YIELD CURVE

The **yield-curve** is the relation between maturity and yield. The *normal* yield curve is one in which the securities with longer maturities are associated with higher yields.



However, *inverted* yield curves do occur, such that shorter-term securities have higher yields.



The inverted yield curve is often a pre-cursor of a recessionary economic period.

The value of a debt security today is the present value of the promised future cash flows, which are the interest and the maturity value. Therefore, the present value of a debt is the sum of the present value of the interest payments and the present value of the maturity value. To calculate the value of a debt security, we discount these future cash flows at some rate that reflects both the time value of money and the uncertainty of receiving these future cash flows. We refer to this discount rate as the **yield**. The more uncertain the future cash flows, the greater the yield. It follows that the greater the yield, the lower the value of the debt security.



YIELD TERMINOLOGY

- **Yield-to-maturity (YTM):** the average annual return assuming the bond is held to maturity.
- **Yield-to-call:** annual return if the bond is called at a specified point in time (and price).
- **Effective annual return:** the return over a year considering the interest and the change in price over the year.
- **Horizon yield:** the return calculated for a specified horizon, future yield, and reinvestment rate.
- **Current yield:** annual interest divided by the current price.

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EXAMPLE: HORIZON YIELD

Consider bonds that have a 10 percent coupon rate, matures in 10 years, and is currently priced at 95. What is the realized yield in two years if the reinvestment rate is 7.5 percent and the yield in two years is 7 percent?

The current price of the bond is \$950 or 95. This is the present value, PV.

The future value of the interest is \$211.53 or 21.153
 [PMT=\$50,n=4,i=3.75 percent]

The future value of the bond is \$1,181.41 or 118.141
 [PMT=\$50,n=16,FV=\$1,000,i=3.5]

The future value with reinvested cash flows is
 $\$211.53 + 1,181.41 = \$1,392.94$ or 139.294

The number of periods is 4.

Solving for the yield:

PV = 950

FV = 1392.94

N = 4

Solve for i

i = Six-month yield = 10.04 percent

Horizon yield is 10.04 percent x 2 = **20.08 percent**

he uncertainty of the bond's future cash flows is affected, in part, by whether the bond is secured or unsecured. A **secured bond** is back by the legal claim to specific property, whereas the **unsecured bond** is backed only by the general credit of the borrower. Unsecured bonds are also referred to as **debentures**. A **subordinated debenture** is an unsecured bond that is junior to senior unsecured bonds, and hence there is more uncertainty pertaining to these securities.

In Wall Street terminology, the term **yield-to-maturity** is used to describe an annualized yield on a security if the security is held to maturity. This is the standard for quoting a market yield on a security. For example, if a bond has a return of 5 percent over a six-month period, the annualized yield-to-maturity for a year is 2 times 5 percent or 10 percent. In the valuation of a bond that pays interest semi-annually – which includes most U.S. corporate bonds –

the discount rate is the six-month yield (that is, the yield-to-maturity divided by 2).

The present value of the maturity value is the present value of a future amount. In the case of a straight coupon security, the present value of the interest payments is the present value of an annuity. In the case of a zero-coupon security, the present value of the interest payments is zero, so the present value of the debt is the present value of the maturity value.

We can rewrite the formula for the present value of a debt security using some new notation and some familiar notation. Because there are two different cash flows -- interest and maturity value -- let PMT represent the coupon payment promised each period and FV represent the maturity value. Also, let N indicate the number of periods until maturity, t indicate a specific period, and i indicate the yield. The present value of a debt security, PV, is:

$$PV = \sum_{t=1}^N \frac{PMT_t}{(1+i)^t} + \frac{FV}{(1+i)^N}$$

If the bond pays interest semi-annually, then N is the number of six-month periods until maturity and i is the six-month yield, or yield-to-maturity ÷ 2.

EXAMPLE: VALUATION OF A STRAIGHT BOND

Problem

Suppose a bond has a \$1,000 face value, a 10 percent coupon (paid semi-annually), five years remaining to maturity, and is priced to yield 8 percent. What is its value?

Solution

Given information:

Maturity value = FV = \$1,000
 Periodic cash flow = PMT = \$100/2 = \$50
 Number of periods = N = 5 x 2 = 10
 Discount rate = i = 8 percent / 2 = 4 percent

$$PV = \sum_{t=1}^{10} \frac{\$50}{(1 + 0.04)^t} + \frac{\$1,000}{(1 + 0.04)^{10}}$$

= \$405.55 + \$675.56 = \$1,081.11

Consider the following example. Suppose that the bond of the Wilma Company has four years remaining to maturity, a coupon rate of 5 percent, and is priced to yield 6 percent. What is the value of a Wilma bond? The value is \$964.90:

TI-83/84 Using TVM Solver	HP10B	Microsoft Excel®
N = 8	1000 FV	=PV(.03,8,25,1000)*-1
i = 3	3 i/YR	
PMT = 25	8 n	
FV = 1000	25 PMT	
Solve for PV	PV	

EXAMPLE: ANNUAL V. SEMI-ANNUAL INTEREST

Consider a \$1,000 face value bond with a coupon rate of 6 percent, matures in 5 years, and is priced to yield 7 percent.

If the bond pays interest annually, then:

FV	=	\$1,000
PMT	=	6 percent of \$1,000, or \$60 per year
N	=	5 years
i	=	7 percent

$$PV = \sum_{t=1}^5 \frac{\$60}{(1+0.07)^t} + \frac{\$1,000}{(1+0.07)^5} = \$959$$

You can use your financial calculator to solve for this by inputting the four known values (FV=1000; PMT=60, N=5, i = 7) and solving for the unknown PV.

If the bond pays interest semi-annually, then:

FV	=	\$1,000
PMT	=	6 percent of \$1,000, divided by 2, or \$30 per year
N	=	10 six-month periods
i	=	7 percent/2, or 3.5 percent

$$PV = \sum_{t=1}^{10} \frac{\$30}{(1+0.035)^t} + \frac{\$1,000}{(1+0.035)^{10}} = \$958$$

You can use your financial calculator to solve for this by inputting the four known values (FV=1000; PMT=30, N=10, i = 3.5) and solving for the unknown PV.

The small difference between the two present values is due to the extra compounding available for reinvestment in the case of a semi-annual bond. Most U.S. bonds pay semi-annual interest and therefore you should assume that all bonds have semi-annually compounding unless told otherwise.

This bond is a *discount bond* because the yield-to-maturity, the 7 percent, is greater than the coupon of 6 percent.

Note: You should assume all bonds are semi-annual pay bonds unless told otherwise.

Try it: Bond values

1. Suppose a bond is priced to yield 6 percent, with a maturity in five years and a coupon rate of 5 percent. What is this bond's quoted value?
2. Suppose a bond matures in six years, has a coupon rate of 6 percent, and is priced to yield 7 percent. What is this bond's quote?
3. Suppose a zero coupon bond matures in ten years. If this bond is priced to yield 10 percent, what is its quoted value?

A. Calculating the yield on a bond

We calculate the yield on a bond using the information about the bond's:

1. Maturity, which indicates the number of periods remaining, N;

2. Coupon, which indicates the cash flow, PMT;
3. Current price, which indicates the value of the bond today, PV; and
4. Face value, which indicates the cash flow at the end of the bond's life, FV.

In other words, we have five elements in a bond valuation, and in the case of solving for the yield to maturity we are given four of the five elements. We cannot solve directly for the yield - - the solution is determined using iteration. Fortunately, our financial calculators and spreadsheets can do this for us.

Suppose a bond with a 5 percent coupon (paid semi-annually), five years remaining to maturity, and a face value of \$1,000 has a price of \$800. What is the yield to maturity on this bond?

Given:¹

Periodic cash flow	= PMT =	\$25
Number of periods	= N =	10
Maturity value, M	= FV =	\$1,000
Present value	= PV =	\$800

The yield per six months is 5.1 percent. Therefore, the yield to maturity is 5.1 percent x 2 = 10.2 percent.

Another way of describing a bond is to use the **bond quote method**. In

TI-83/84 Using TVM Solver	HP10B	Microsoft Excel®
N = 10	1000 FV	=RATE(10,25,-800,1000,0) * 2
PV = 800	800 +/- PV	
PMT = 25	10 n	
FV = 1000	25 PMT	
<i>Solve for i</i>	I	
<i>Then multiply by 2</i>	X 2	

BOND QUOTES

A bond may be stated in terms of a percentage of the bond's face value or as a dollar value. Consider a bond with a face value of \$1,000 and a coupon rate of 6 percent. We can value this bond using the percentages or the dollar values:

	As a percentage	As a dollar value
Face value	100	\$1,000
Coupon payments	3 every six months	\$30 every six months

The advantage of using the percentage method (a.k.a. bond quote method) is that you don't have to know the bond's face value to calculate the yield or value - you state every parameter as a percentage of the face value.

this case, the interest payment and the value of the bond are stated as a percentage of the bond's face value. This is the method that you will see most often used in practice. For example, if a bond is quoted at 115, this means that it is trading at 115 percent of its face value. If it has a face value of \$500, it is therefore priced at \$575. A bond quote of 85 indicates that the bond is trading for 85 percent of its face value. The bond quote method provides a more generic method of communicating a bond's value - you don't have to know the bond's face value to

determine its price.

Continuing this same problem, restating the elements in terms of the bond quote,

Periodic cash flow	= PMT =	2.5
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¹ Hint: The bond is selling at a discount, so the YTM must be greater than the coupon rate of 5 percent

Number of periods	= N	=	10
Maturity value, M	= FV	=	100
Present value	= PV	=	80

TI-83/84 Using TVM Solver	HP10B	Microsoft Excel®
N = 10	100 FV	=RATE(10,2.5,-80,100,0) * 2
PV = 80	80 +/- PV	
PMT = 2.5	10 n	
FV = 100	2.5 PMT	
Solve for i	I	
Then multiply by 2	X 2	

When we solve for the yield to maturity, we simply use 100 for the face value or FV. We solve the problem in the same manner as before.

Try it: Bond yields

1. Suppose a bond is priced at 98, with a maturity in five years and a coupon rate of 5 percent. What is this bond's quoted value?
2. Suppose a bond matures in six years, has a coupon rate of 6 percent, and is quoted at 101. What is this bond's yield to maturity?
3. Suppose a zero coupon bond matures in ten years. If this bond is priced at 65, what is its yield to maturity?

B. The yield curve

The **yield curve** is the set of spot rates for different maturities of similar bonds. The normal yield curve is upward-sloping, which means that longer maturity securities have higher rates. Term structure theories are explanations for the shape of the yield curve:

- Expectations hypothesis
- Liquidity preference hypothesis
- Segmented market hypothesis

The **expectations hypothesis** states that current interest rates are predictors of future interest rates (that is, "forward" rates). In other words, a spot rate on a two-year security (R_2) should be related to the spot rate on a one-year security (R_1) and the one-year forward rate one year from now (${}_1r_1$):²

$$(1 + R_2)^2 = (1 + R_1)(1 + {}_1r_1)$$

If the one-year rate is 5 percent and the two year rate is 6 percent, the expectations hypothesis implies that the one-year rate *one year from today* is the rate that solves the following:

$$(1 + 0.06)^2 = (1 + 0.05)(1 + {}_2r_1)$$

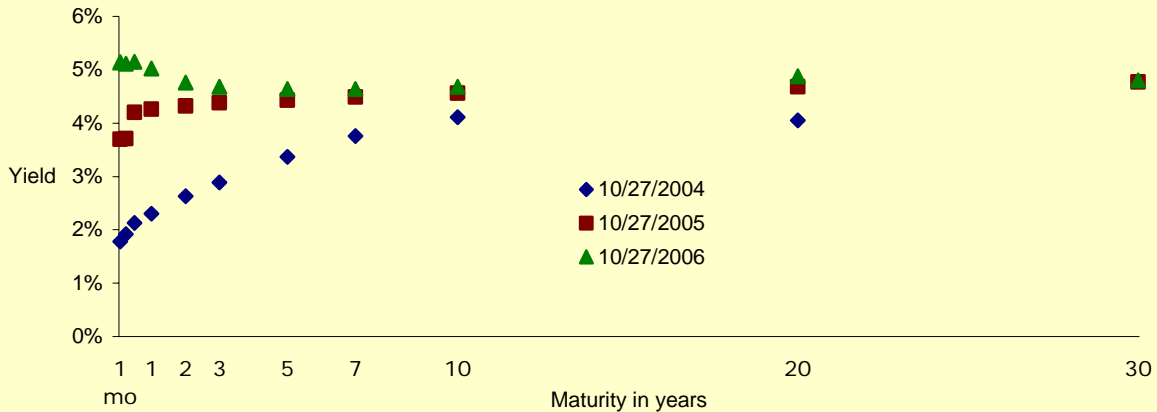
Using Algebra, we see that the one year rate for next year that is inferred from the current rates is 7.01 percent:

² You'll notice in this section that we are using an upper-case "R" to indicate the spot rate and a lower case "R" to indicate the forward (i.e., future) rate.

$$1.1236 = 1.05 (1 + {}_2r_1)$$

$${}_2r_1 = 7.01 \text{ percent}$$

YIELD CURVES: COMPARISON OF OCTOBER 2006 WITH OCTOBER 2004 AND OCTOBER 2005



Source of data: Yahoo! Finance and the U.S. Treasury

- The yield curve in October 2004 was a "normal" curve.
- The yield curve in October 2005 was a flattening curve.
- The yield curve in October 2006 was an inverted curve.

The **liquidity preference hypothesis** states that investors prefer liquidity and therefore require a premium in terms of higher rates if they purchase long-term securities. The segmented market hypothesis states that there are different preferences for different segment of the market and that the yield for a given maturity is dependent on the supply and demand of securities with that maturity.

No matter the explanation for the shape of the yield curve, the most accurate valuation of a bond considers the yield curve. We can consider the yield curve when we use spot rates for different maturities in the bond valuation:

$$P_0 = \sum_{t=1}^N \frac{CF_t}{(1+i_t)^t}$$

where

i_t = discount rate for the period t

Consider annual-pay bond with a coupon of 5 percent, a face value of \$1,000, and four years to maturity. Suppose the yield curve indicates the following spot rates:³

Maturity	Spot rate
1 year	4.0 percent
2 year	4.5 percent
3 year	5.0 percent
4 year	5.5 percent

³ A **spot rate** is a current rate.

What is this bond's value? To determine this value, we discount the individual cash flows at the appropriate spot rate and then sum these present values.

<u>Maturity</u>	<u>Spot rate</u>	<u>Bond cash flow</u>	<u>Present value of cash flow⁴</u>
1 year	4.0 percent	\$50	\$48.077
2 years	4.5 percent	\$50	45.786
3 years	5.0 percent	\$50	43.192
4 years	5.5 percent	\$1,050	<u>847.578</u>
			\$984.633

Using the yield curve, the value of the bond is \$984.633. If, on the other hand, we had simply used the four-year spot rate, we the value of the bond is \$982.474. The extent of the difference depends on the slope of the yield curve

C. Option-like features

The issuer may add an option-like feature to a bond that will either provide the issuer or the investor more flexibility and/or protection. For example, a **callable bond** is a bond that the issuer can buy back at a specified price. This option is a *call option* (i.e., an option to buy) of the issuer and the investor bears the risk of the bond being called away, especially when interest rates have fallen. The callable bond agreement specifies the price at which the issuer will buy back the bond and there may be a schedule of prices and dates, with declining call prices as the bond approaches maturity. A **puttable bond**, on the other hand, is a bond that gives the investor the right to sell the bond back to the issuer at a pre-determined price, usually triggered by an event, such as a change in control of the issuer. A puttable bond, therefore, gives the investor a *put option* (i.e., an option to sell) on the bond.

The **yield to call** is the yield on a callable bond, considering that the bond is called at the earliest date. Consider the following example. The Bagga Company issued bonds that have five years remaining to maturity, and a coupon rate of 10 percent. These bonds have a current price of 115. These bonds are callable starting after two years at 110. What is the yield-to-maturity on these bonds? What is the yield-to-call on these bonds? The first step is to identify the given information:

<u>Given parameter</u>	<u>Yield to maturity</u>	<u>Yield to call</u>
FV	100	110
PV	115	115
PMT	5	5
N	10	4

For the yield to maturity,

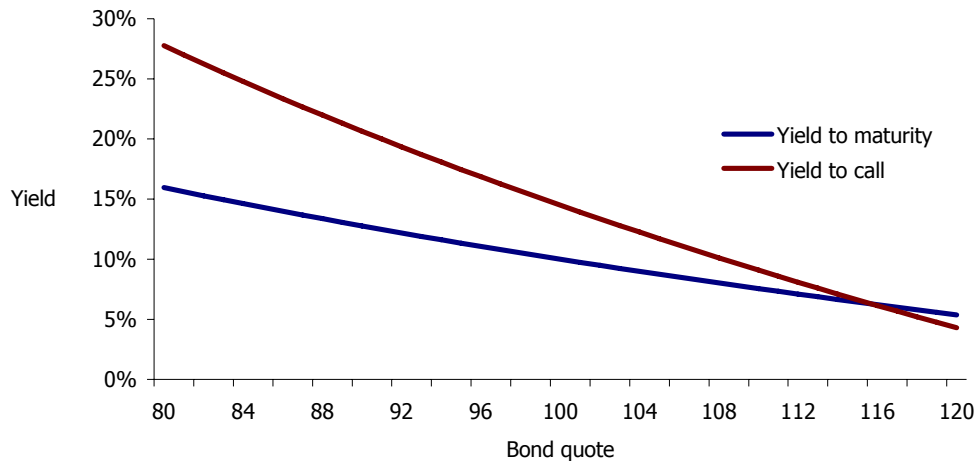
TI-83/84 Using TVM Solver	HP10B	Microsoft Excel®
N = 10	100 FV	=RATE(10,5,-115,100,0) * 2
PV = 115	115 +/- PV	
PMT = 5	10 n	
FV = 100	5 PMT	
Solve for i	I	
Then multiply by 2	X 2	

⁴ This is calculated simply as the present value of the lump-sum future value. For example, for the cash flow of \$50 three years from now, the present value is $\$50 / (1 + 0.05)^3 = \43.192

The yield to maturity is 6.4432 percent. For the yield to call,

TI-83/84 Using TVM Solver	HP10B	Microsoft Excel®
N = 4	110 FV	=RATE(4,5,-115,110,0) * 2
PV = 115	115 +/- PV	
PMT = 5	4 n	
FV = 110	5 PMT	
Solve for i	I	
Then multiply by 2	X 2	

The yield to call is $3.3134 \times 2 = 6.6268$ percent. We can see the relation between these yields on the bond's current value (that is, the PV) in bond quote terms:



Both the yield to call and the yield to maturity are lower for higher current bond values.

Another option-like feature is a conversion feature. A **convertible bond** has such a feature, which gives the investor the right to exchange the debt for a specified other security of the issuer, such as common stock. The exchange rate is specified in the convertible bond agreement.

The valuation of a bond with option-like features is quite complex because it involves valuing the option as well. This is beyond the scope of this module.

D. Bond ratings

A **bond rating** is an evaluation of the default risk of a given debt issue by a third party, a ratings service. There are three major ratings services: **Moody's**, **Standard & Poor's**, and **Fitch**. Ratings range from AAA to D, with some further ratings within a class indicated as + and - or with numbers, 1, 2 and 3. The top four ratings classes (without counting breakdowns for +,- or 1,2,3) indicate **investment-grade** securities. **Speculative grade bonds** (a.k.a. **junk bonds**) have ratings in the next two classes. C-rated bonds are income or revenue bonds, trading flat (in arrears), whereas D-rated bonds are in default.

Ratings are the result of a fundamental analysis of a bond issue, assessing the default risk of the issue. Ratings are affected by many factors, including:

- profitability (+)
- size (+)
- cash flow coverage (+)

- financial leverage (-)
- earnings instability (-)

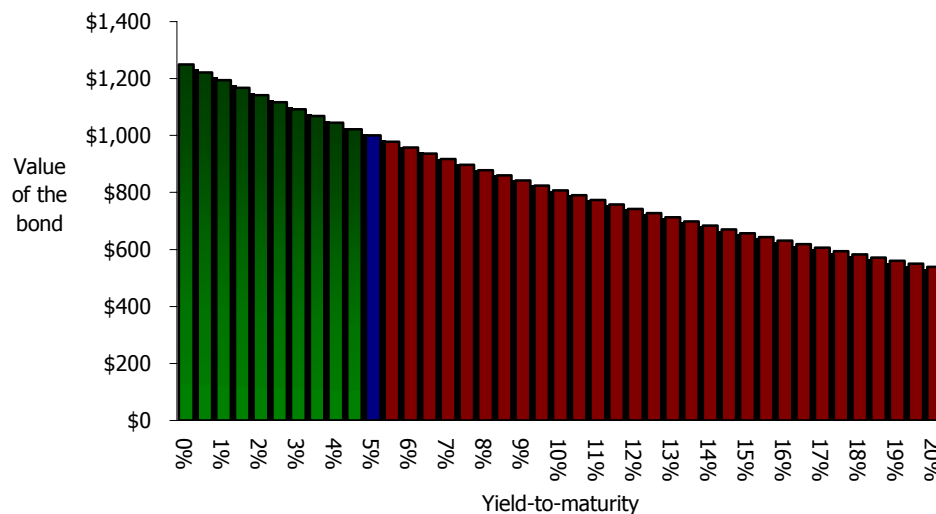
Ratings most often are the same across the rating agencies, but **split ratings** do occur. Ratings are reviewed periodically and may be revised upward or downward as the financial circumstances of the issuer change. For a given issuer, ratings are performed on the most senior unsecured issue and then junior issues are rated (generally at a lower rating) according to their indentures. Because bond ratings are of specific issues of an issuer, it is possible for a given issuer to have bonds that are rated differently. The primary differences relate to maturity and security on the particular issue.

E. The coupon-yield relationship

When we look at the value of a bond, we see that its present value is dependent on the relation between the coupon rate and the yield.

- If the coupon is less than the yield to maturity, the bond sells at a discount from its face or maturity value.
- If the coupon is greater than the yield to maturity, the bond sells at a premium to its face value.
- If the coupon is equal to the yield to maturity, the bond sells at par value (its face value).

Consider a bond that pays 5 percent coupon interest semi-annually, has five years remaining to maturity, and a face value of \$1,000. The value of the bond depends on the yield to maturity: the greater that yield to maturity, the lower the value of the bond. For example, if the yield to maturity is 10 percent, the value of the bond is \$806.96. If, on the other hand, the yield to maturity is 4 percent, the value of the bond is \$1,044.91.



You can see that there is convexity in the relation between the value and the yield. In other words, the relation is not linear, but rather *curvilinear*.

- Coupon rate > Yield to maturity ⇨ Premium
- Coupon rate < Yield to maturity ⇨ Discount
- Coupon rate = Yield to maturity ⇨ Face value

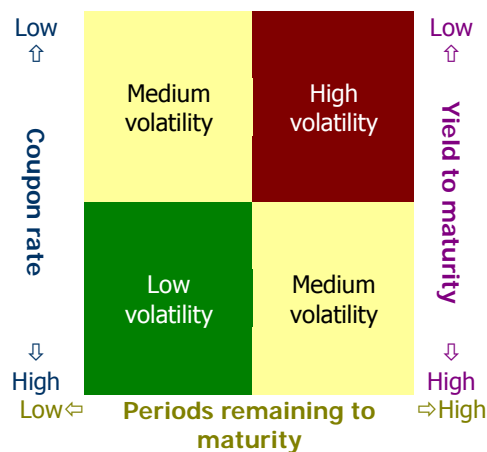
F. Duration

The pattern of cash flows and the time remaining to maturity all relate to the sensitivity of a bond's price to changes in yields. Also, as we saw in the earlier section, the sensitivity of a bond's price depends on the size of the yield because of the curvilinear relation between yield and value. In general,

- The greater the coupon rate, the lower the sensitivity to changing interest rates, *ceteris paribus*.
- The greater the time remaining to maturity, the greater the sensitivity to changing interest rates, *ceteris paribus*.
- The greater the yield to maturity, the lower the sensitivity to changing interest rates.

The convex relation between value and yield therefore means that we need to consider what the starting yield is as we consider the effect of changes in yields on the bond's value.

An implication of this convex relationship is that the change in a bond's value depends on what the starting yield is, how much it changes, and whether it is an up or down change in yield. That's where **duration** comes in – it's a measure of the average length of time for the bond's cash flows and is used to estimate the change in price of the bond for a change in yield.



Basically, duration is a time-weighted measure of the length of a bond's life. The longer the duration, the greater the bond's volatility with respect to changes in the yield to maturity.

There are different measures of duration for different purposes: Macauley duration, modified duration, and effective duration.

Macauley's duration is the percentage change in the value of the bond from a small percentage change in its yield-to-maturity. We calculate this measure of duration using a time-weighting of cash flows.

$$\text{Macauley's duration} = \frac{\text{Present value of time weighted cash flow}}{\text{Present value of the bond}}$$

The weight is the period. For example, if interest is paid annually, the weight for the first interest payment is 1.0, the weight for the second interest payment is 2.0, and so on.

Modified duration is a measure of the average length of time of the bond's investment, considering that some cash flows are received every six months and the largest cash flow (the face value) is received at maturity. Modified duration requires an adjustment to Macauley's duration:

$$\text{Modified duration} = \frac{\text{Macauley's duration}}{(1 + \text{yield-to-maturity})}$$

If interest is paid semi-annually, the time weighting in the Macauley duration measures uses the whole and half years (0.5, 1.0, 1.5, 2.0, etc.) and the yield to maturity in the modified duration's denominator is the semi-annual rate (i.e., yield to maturity ÷ 2).

Effective duration is a measure of the impact on a bond's price of a change in yield-to-maturity. Though similar to Macauley's duration in interpretation, its calculation is flexible to allow it to be used in cases when the bond has an embedded option (e.g., a callable bond).

$$\text{Effective duration} = \frac{PV_- - PV_+}{2 PV_0 \Delta i}$$

where

Δi = change in yield

PV_+ = Value of the bond if the yield went up by Δi

PV_- = Value of the bond if the yield went down by Δi

PV_0 = Value of the bond at the yield-to-maturity

The approximate percentage change in price for both the modified and effective duration measures is:

$$\% \text{ change} = -1 \times \text{duration} \times \text{change in yield}$$

We say that this is an approximate change because we still haven't accounted for the **convexity**, or the curvature in the relationship. There is a measure of convexity that can be used to fine-tune this approximate price change to get a closer estimate of the change, but this calculation is outside of the scope of this module.

Consider a bond with a 10 percent coupon, five years to maturity, and a current price of \$1,000. What is the duration of this bond? The modified duration is 3.8609 years.

Period	Cash flow	Present value	Time-weighted cash flow
0.5	\$50	\$47.62	\$23.81
1.0	\$50	\$45.35	45.35
1.5	\$50	\$43.19	64.79
2.0	\$50	\$41.14	82.27
2.5	\$50	\$39.18	97.94
3.0	\$50	\$37.31	111.93
3.5	\$50	\$35.53	124.37
4.0	\$50	\$33.84	135.37
4.5	\$50	\$32.23	145.04
5.0	\$1,050	\$644.61	<u>3,223.04</u>
Sum			\$4,053.91

$$\text{Macauley duration} = \frac{\$4,053.91}{\$1,000} = 4.0539$$

$$\text{Modified duration} = \frac{4.0539}{1.05} = 3.8609$$

The value of the bond if the yield is 9 percent is \$1,039.56, whereas the value of the bond if the yield is 11 percent is \$962.31. If the current yield is 10 percent, resulting in a current value is \$1,000, the effective duration is 3.8626 years:

$$\text{Effective duration} = \frac{\$1,039.56 - 962.31}{2(\$1000)(0.01)} = \frac{77.25}{20} = 3.8625 \text{ years}$$

This means that if we expect yields to increase 2 percent, the expected change in this bond's price is

$$\% \text{ change} = -1 \times \text{duration} \times \text{change in yield}$$

$$\% \text{ change} = -1 (3.8625) (-0.02) = -7.725 \text{ percent}$$

Let's see how accurate this is. If the yield goes from 10 percent to 8 percent, the bond's value goes from \$1,000 to \$1,081.11, a change of 8.111%. Why didn't we hit the price on the mark? Two reasons: (1) the estimate using duration is good for very small changes in yields and less accurate for large changes, and (2) we haven't considered the convexity.

Why worry about duration? Because we are interested in measuring and managing risk. For example, if we put together a portfolio of bonds we are interested in the risk of that portfolio, which is affected in part by the interest-sensitivity of the individual bonds that comprise the portfolio.

EXAMPLE: DURATION

Consider an annual-pay bond with a face value of \$1,000, four years remaining to maturity, a coupon rate of 8 percent, and a yield of 6 percent.

Macauley and Modified Duration

Period	Cash flow	Present value of cash flows (discounted at 6 percent)	Time-weighted cash flow (period x cash flow)	Present value of time-weighted cash flow
1	\$ 80.00	\$ 75.47	\$ 80.00	\$ 75.47
2	80.00	71.20	160.00	142.40
3	80.00	67.17	240.00	201.51
4	1,080.00	\$ <u>855.46</u>	4,320.00	<u>3,421.84</u>
		\$1,069.30		\$ 3,841.22

$$\text{Macauley's duration} = \$3,841.22 / \$1,069.30 = \mathbf{3.5923}$$

$$\text{Modified duration} = 3.5923 / 1.06 = \mathbf{3.3889}$$

Approximate percentage price change for an increase in yield of 1 percent:
% change = $-(3.3889)(0.01) = \mathbf{-3.3889 \text{ percent}}$

Effective Duration

Yield-to-maturity	Present value of the bond
5 percent	\$1,106.38
6 percent	\$1,069.30
7 percent	\$1,033.87

$$\text{Effective duration} = (\$1,106.38 - \$1,033.87) / [2 (\$1,069.30)(0.01)] = \mathbf{3.3904}$$

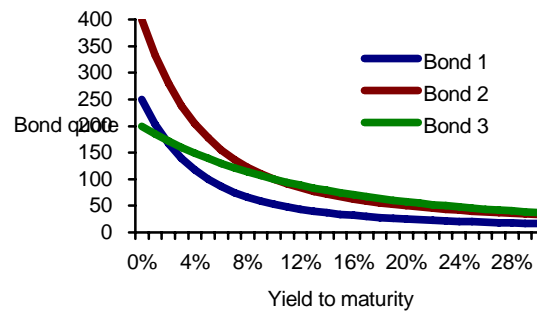
Approximate percentage price change for an increase in yield of 1 percent:

$$\% \text{ change} = - (3.3904)(0.01) = \mathbf{-3.3904 \text{ percent}}$$

Convexity is the degree of curvature of a bond's value-YTM relationship. We use convexity to refine our estimate of the bond's sensitivity to changes in the YTM. Though the mathematics of convexity is out of the scope of this course, you should understand the concept of convexity and the fact that different bonds may have different convexity. Consider three bonds, labeled 1, 2 and 3.

Bond	Coupon	Maturity
1	5 percent	30 years
2	10 percent	30 years
3	10 percent	10 years

These three bonds have different convexity by virtue of their different coupons and maturities:



Try it: Duration

- Consider a bond that has a coupon rate of 5 percent, five years remaining to maturity, and is priced to yield 4%. Assume semi-annual interest.
 - What is the effective duration for this bond?
 - What is the approximate change in price if the yield increases from 4% to 5%?
- Consider a bond that has a coupon rate of 5 percent, ten years remaining to maturity, and is priced to yield 4%. Assume semi-annual interest.
 - What is the effective duration for this bond?
 - What is the approximate change in price if the yield increases from 4% to 5%?

G. Summary

In this module, we look at bond valuation and the sensitivity of the bond's valuation to changes in interest rates. We explore valuation issues beyond the simple value of a straight coupon bond, extending the valuation to a non-flat yield curve. We also take a brief look at bond ratings and the determinants of these ratings, as well as the option-like features of bonds. Further, we look at the measures of interest rate sensitivity: Macauley, modified, and effective duration measures. These measures help us gauge the sensitivity of a bond's value to changes in yields.

2. Learning outcomes

- | | |
|--------|--|
| LO9.1 | List the features of bonds that result in different interest rate patterns and how these features affect a bond's valuation. |
| LO9.2 | Calculate the yield to maturity, horizon yield, and yield to call for a bond. |
| LO9.3 | Distinguish between the value of a bond with annual interest v. semi-annual interest. |
| LO9.4 | List and explain briefly the different explanations for the shape of the yield curve. |
| LO9.5 | Calculate the value of a bond using the yield curve. |
| LO9.6 | Explain how option-like features affect a bond's value. |
| LO9.7 | Distinguish between an investment grade debt security and a junk bond in terms of ratings. |
| LO9.8 | Demonstrate through calculations the relation between a bond's value and its yield to maturity. |
| LO9.9 | Calculate the Macauley, modified, and effective duration of a bond and the expected change in a bond's value for a given change in yield. |
| LO9.10 | Explain and demonstrate how the estimated change in a bond's value using duration measures does not predict the change in value precisely. |

3. Module Tasks

A. Required readings

- Chapter 17, "Bond Yields and Prices," *Investments: Analysis and Management*, by Charles P. Jones, 9th edition.
- Chapter 18, "Bond Analysis and Strategy," *Investments: Analysis and Management*, by Charles P. Jones, 9th edition.

B. Other material

- [Bond Center Education](#), Yahoo! Finance

C. Optional readings

[Valuation of Corporate Securities](#), by StudyFinance.com
[Duration](#), by Financial Pipeline

D. Practice problems sets

- [Textbook author's practice questions, with solutions.](#)
- [StudyMate Activity](#)
- [Investment Mini-Quiz: Bonds](#), by Learning for Life
- [Quiz: Bonds](#), by Smart Money

E. Module quiz

- Available at the [course Blackboard site](#). See the [Course Schedule](#) for the dates of the quiz availability.

F. Project progress

- At this point, you should have completed all the data gathering and analysis for your project and a great deal of the write-up.
- Focus on your writing, Make sure that all statements are supported with citations, that all of your graphs are derived from your Excel worksheet, and that you have completed all required tasks.

4. What's next?

In this module, we looked at the valuation and risk associated with bonds. In Module 10, we introduce you to derivatives: options, futures, and forwards. Though derivative instrument have been around for a long time, the actual traded options and futures contracts and the way in which investors use these contracts are only a few decades old. Investors use derivatives for both speculation and hedging. Because derivatives derive their value from some other asset, the pricing of derivatives is rather complex.

Solutions to Try it problems

Try it: Bond values

1. Suppose a bond is priced to yield 6 percent, with a maturity in five years and a coupon rate of 5 percent. What is this bond's quoted value?

$$I = 3\%; N = 10; PMT = 2.5; FV = 100$$

$$\text{Solve for PV: } PV = \mathbf{95.734899}$$

2. Suppose a bond matures in six years, has a coupon rate of 6 percent, and is priced to yield 7 percent. What is this bond's quote?

$$I = 3.5\%; N = 12; PMT = 3; FV = 100$$

$$\text{Solve for PV: } PV = \mathbf{95.16833}$$

3. Suppose a zero coupon bond matures in ten years. If this bond is priced to yield 10 percent, what is its quoted value?

$$I = 5\%; N = 20; PMT = 0; FV = 100$$

$$\text{Solve for PV: } PV = \mathbf{37.68895}$$

Try it: Bond yields

1. Suppose a bond is priced at 98, with a maturity in five years and a coupon rate of 5 percent. What is this bond's quoted value?

$$PV = 98; N = 10; PMT = 2.5; FV = 100$$

$$\text{Solve for } i: I = 2.73126;$$

$$YTM = \mathbf{5.46251\%}$$

2. Suppose a bond matures in six years, has a coupon rate of 6 percent, and is quoted at 101. What is this bond's yield to maturity?

$$N = 12; PMT = 3; PV = 101; FV = 100$$

$$\text{Solve for } i: I = 2.90013;$$

$$YTM = \mathbf{5.80027\%}$$

3. Suppose a zero coupon bond matures in ten years. If this bond is priced at 65, what is its yield to maturity?

$$N = 20; PV = 65; FV = 100; PMT = 0$$

$$\text{Solve for } i: I = 2.177279\%;$$

$$YTM = \mathbf{4.35456\%}$$

Try it: Duration

1. Consider a bond that has a coupon rate of 5 percent, five years remaining to maturity, and is priced to yield 4%. Assume semi-annual interest.

- a. What is the effective duration for this bond?

Yield	Value
3%	109.22218
4%	104.49129
5%	100.00000

$$\text{Effective duration} = (109.22218 - 100) / (2 (104.49129) (0.01)) = \mathbf{4.41289}$$

- b. What is the approximate change in price if the yield increases from 4% to 5%?

$$4.41289 \times 0.01 \times -1 = \mathbf{-4.41289\%}$$

2. Consider a bond that has a coupon rate of 5 percent, ten years remaining to maturity, and is priced to yield 4%. Assume semi-annual interest.

- a. What is the effective duration for this bond?

Yield	Value
3%	117.16864
4%	108.17572
5%	100.00000

$$\text{Effective duration} = (117.16864 - 100) / (2 (108.17572) (0.01)) = \mathbf{7.935533}$$

- b. What is the approximate change in price if the yield increases from 4% to 5%?

$$7.935533 \times 0.01 \times -1 = \mathbf{-7.935533\%}$$

Note: You should get the identical effective duration and price change if you use the dollar value of the bonds, assuming a \$1,000 face value.