Module 7
Asset pricing models

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1. Overview

Asset pricing models are different ways of interpreting how investors value investments. Most models are based on the idea that investors hold well-diversified portfolios and that investors are rational.

The most widely known asset-pricing model is the **capital asset pricing model (CAPM)**. This model results in a simple view of how assets are valued: investors are rewarded, in terms of greater expected return, if they bear greater market risk.

An alternative to the CAPM is the **arbitrage pricing theory (APT)**, which is based on different assumptions regarding how markets and investors behave. The APT results in a more general view that returns are explained by unexpected changes in fundamental economic factors, such as changes industrial production.

A. The Capital Asset Pricing Model

Portfolio theory is the foundation of asset pricing and concerns how investors make investment decisions. Portfolio theory tells us that investors consider expected return and risk important in pricing assets.

There are two prominent theories that describe the trade-off between risk and return. The first is the capital asset pricing model (CAPM), which we cover first. The second is the arbitrage pricing theory (APT), which we also cover in this module.

The basic idea of the capital asset pricing theory is that there is a positive relation between risk and expected return and that the only risk that is relevant in an investor's decision is **market risk**, which cannot be removed by diversification (hence, it is often referred to as **non-diversifiable risk**). If investors can hold diversified portfolios, then the only risk that is "priced" (or in other words, rewarded in terms of higher return), is the risk that can<u>not</u> be diversified away.

The CAPM, developed by William Sharpe, extends the work of Harry Markowitz that we looked at in a previous module, by introducing the possibility that investors can borrow or lend at the risk-free rate of interest.[1] Lending at the risk-free rate means that the investor buys a risk-free investment, such as a U.S. Treasury Bill. Borrowing at the risk-free rate means that the investor can borrow at the same rate as the U.S. Treasury Bill. Of course this is not realistic, but it helps us determine what is important in the pricing of assets – and we can add the realism after we understand the basic framework. The risk-free investment and borrowing possibilities expand investors' opportunities.

[1] William F. Sharpe, "A Simplified Model of Portfolio Analysis" *Management Science*, Vol. 9 (January 1963) pp. 277-293.

A **risk-free asset** is one in which there is no correlation of its returns with those of a risky asset.[2] The expected return on the risk-free asset is r_f and the standard deviation of its returns is zero.

The ability to invest in the risk-free asset or borrow at the risk-free rate enhances opportunities:

- More risk-averse investors can lend at the risk-free rate and hold some portion of the market portfolio in their portfolio.
- Less risk-averse investors can borrow at the risk-free rate and leverage the market portfolio.

Think about what a risk-free asset does to a portfolio's risk. The formula for the portfolio risk for a two-security portfolio is:

$$\sigma_p = \sqrt[2]{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{AB}}$$

where

σ_p, σ_A and σ_B	indicate the standard deviations of the portfolio, Security A and Security B, respectively;
w_A and w_B	indicate the weight in the portfolio of A and B;[3] and
ρ_{AB}	indicates the correlation between A and B.

If Security A is risk-free, this means that the σ_A is zero and that ρ_{AB} is zero. This also means that the portfolio risk is:[4]

$$\sigma_p = \sqrt[2]{w_B^2\sigma_B^2} = w_B\sigma_B$$

From this equation, you can see that the risk of the portfolio then depends on how much is invested in the risky asset and the risk of that risky asset.

Consider the following example. Suppose you have two securities, K and L, that have a correlation of 0.30 and the following characteristics:

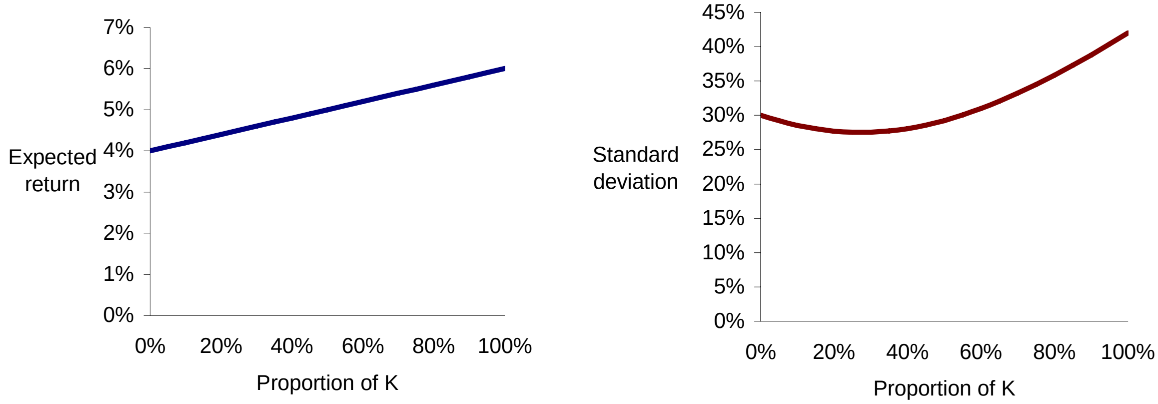
Security	**Expected return**	**Standard deviation**
K	6%	42%
L	4%	30%

The expected return and standard deviation of the two-security portfolio in different proportions of K and L (with K's weight along the horizontal axis), is:

[2] The risk that it is free of is default risk. Such a security may still have other types of risk, such as interest rate risk and reinvestment rate risk.

[3] Note that the weights must sum to 1.0.

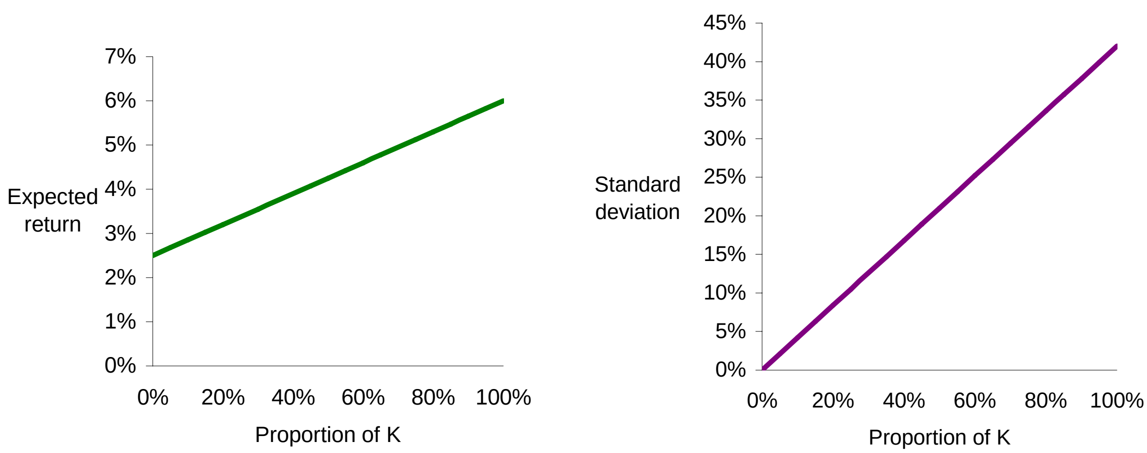
[4] This is because both of the terms $w_A^2\sigma_A^2$ and $2w_A w_B \sigma_A \sigma_B \rho_{AB}$ become zero when σ_A is zero.



But if we were to combine security K with a risk-free security, M:[5]

Security	Expected return	Standard deviation
K	6%	42%
M	4%	0%

We can see that the return and risk of the portfolio is quite different:



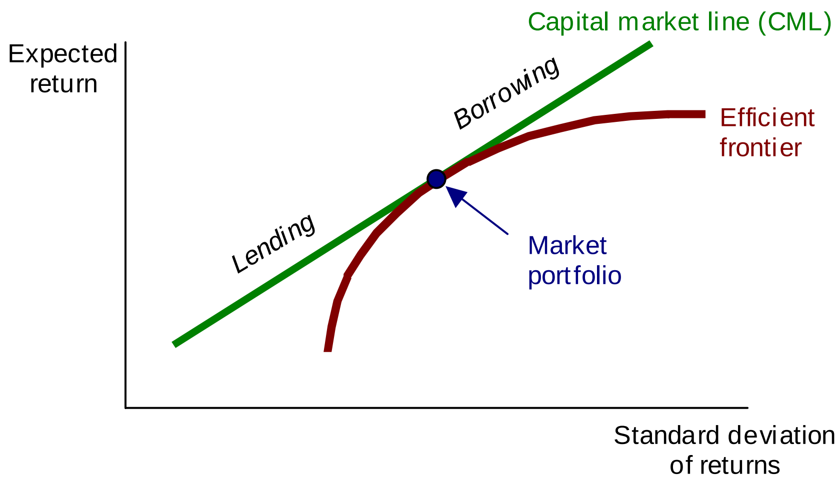
The **separation theorem** states that the investment and financing decisions of an investor are separate:

- The investment decision is the decision to invest in the market portfolio and/or the risk-free asset.

[5] Remember: if the security is risk-free, it's returns do not have any variation and hence there is no standard deviation. Also, if the returns on the risk-free security do not vary, then there would be no correlation with the returns of the other security, K.

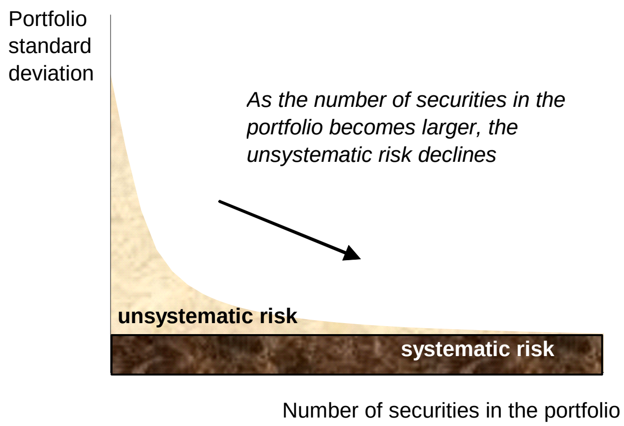
- The financing decision is whether to lend of borrow along the capital market line (CML).

The **capital market line** is the set of possible optimal portfolios that exit once we introduce the risk-free asset. The capital market line is the line that is tangent to the efficient frontier (that we developed in Module 6):



The **market portfolio** is the point of tangency of the CML and the efficient frontier and is comprised of all investable risky assets. The market portfolio is a completely diversified portfolio. A completely diversifiable portfolio has no unsystematic risk. The only risk in a diversified portfolio is **systematic risk.**

In the often-used diagram of possible portfolios of different sizes, as shown below, we see that the unsystematic, diversifiable risk disappears for portfolios of a sufficiently large number of securities; however, the non-diversifiable, systematic risk remains:[6]



RISK SEMANTICS

Risk that goes away when a portfolio is diversified:
- Unsystematic risk
- Unique risk
- Diversifiable risk
- Firm-specific risk

Risk that cannot be diversified away:
- Systematic risk
- Non-diversifiable risk
- Market risk

[6] This graph is constructed by calculating the average portfolio standard deviation for each size portfolio. Therefore, for the average portfolio standard deviation of portfolios of the size of one security is plotted, followed by the average portfolio standard deviation of portfolios of the size of two securities, etc. Because of diversification, the risk of a portfolio declines for ever-larger portfolios – to a point.

B. The capital asset pricing model

The **capital asset pricing model** (CAPM) is a theory in which the expected return on an asset is the sum of the return on a risk-free asset and the return commensurate with the asset's *market risk*:

$$E(r_i) = E(r_f) + \underbrace{\overbrace{E(r_m - r_f)}^{\text{risk premium for the market}}\ \beta}_{\text{asset's risk premium for bearing market risk}}$$

where

$E(r_i)$	= the expected return on asset i,
$E(r_f)$	= the expected return on the risk-free asset,
$E(r_m - r_f)$	= the expected risk premium on the market portfolio, and
β	= beta, the measure of market risk.

Whereas the term $E(r_m - r_f)$ is the risk premium for the market as a whole (i.e., on average for the entire market), the term $E(r_m - r_f)\ \beta$ is, therefore, the compensation for the i^{th} asset's market risk.

RISK PREMIUMS

The risk premium for the market is often confused with the market risk premium for an asset. The difference is the factor of β.

The **risk premium of the market** – that is, for the average stock in the market – is:

$$E(r_m - r_f)$$

The **market risk premium for an individual asset** is the risk premium for the market, adjusted by the factor of β:

$$E(r_m - r_f)\ \beta$$

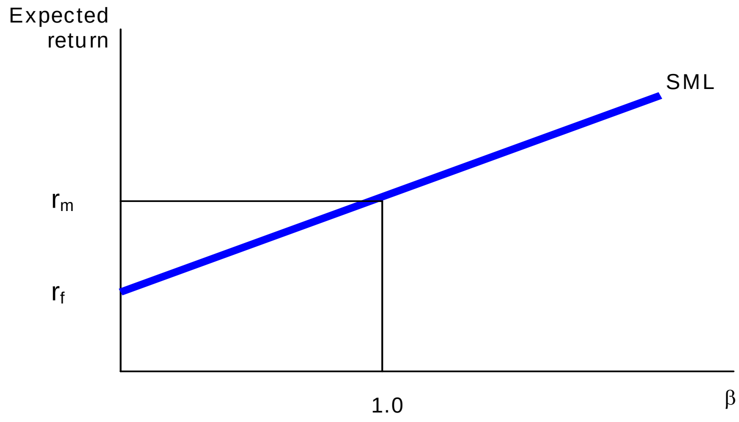
The model makes a many assumptions about the market and how investors view investments:

- Investors base their investment decision on expected return and the variance of returns.
- Investors prefer more wealth to less wealth.
- Any amount can be borrowed or lent at the risk free rate of interest.
- Investors have homogeneous expectations.
- All investors have the same investment horizon.
- All investments are infinitely divisible.
- There are no taxes or transactions costs.
- There is no inflation.
- The capital market is in equilibrium.
- The market portfolio is an efficient portfolio that contains all investable risky assets.

A number of these assumptions are basically common sense, such as the assumption that investors prefer more wealth to less. And there are some that are not very realistic, such as no taxes (which we may all wish for, but it is not likely to come true). For now, let's make these assumptions and then we'll see what happens if we relax a few of these.

Market risk is the sensitivity of the asset's returns to changes in the return on the market. Because investors are only compensated (in terms of a higher expected return) for bearing market risk, the greater an asset's market risk, the greater the expected return on the asset.

The **security market line** (SML) captures the relation between expected return and market risk (β).



The security market line depicts the following:

- There is a positive relation between market risk and expected return.
- The β of the market is 1.0
- The expected return of an asset with a $\beta = 1$ is r_m.

Beta, β, is a measure of market risk: the greater the beta, the more sensitive are the returns on the stock to changes in the returns on the market. A beta of 1.2 for Company X means that:

- If the market return goes up 1 percent, the return of Company X stock is expected to go up 1.2 times 1 percent, or 1.2 percent.
- if the market in general goes down 1 percent, the return of Company X stock is expected to go down 1.2 percent.

We estimate a stock's beta using the **market model regression**:

$$R_{it} = a + b\, R_{mt}$$

where

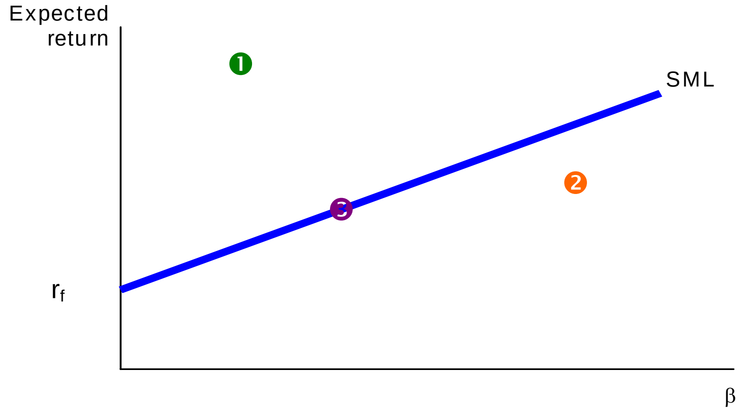
R_{it}	=	the return on stock i on day t,
R_{mt}	=	the return on the market proxy for day t,
a	=	the estimated intercept, and
b	=	the estimated slope, the estimate for b.

We usually estimate this model using sixty monthly returns or at least 250 daily returns. And we often use a comprehensive market index, such as the S&P 500 index, as the proxy for the market portfolio. For a detailed explanation of how to estimate a beta, check out Estimating a market model: Step-by-step.

Suppose that a company's stock has a beta of 1.2. And suppose that the expected risk-free rate of interest is 4 percent and the expected premium for the market as a whole is 5 percent. What is the expected return on this stock?

R_{ABC}	=	0.04 + 1.2 (0.05)
	=	0.04 + 0.06
	=	10 percent

If a security has an expected return that deviates from what is expected based on the SML, we consider the stock to be mispriced. For example, if the expected return is greater than that expected with the SML, the stock is considered underpriced; as the price is adjusted upward to reflect its true value, the return will be greater than that predicted by the SML. Consider the three stocks, labeled 1, 2 and 3 whose expected return and market risk are plotted below:



From this, we conclude that:

- ❶ is underpriced: the expected return is greater than that sufficient for its market risk.
- ❷ is overpriced: the expected return is less than that sufficient for its market risk.
- ❸ is correctly priced: the expected return is sufficient for its market risk.

The **beta of a portfolio** is the weighted average of the betas of the securities in the portfolio, where the weights are the proportion of the portfolio invested in the asset.

A portfolio comprised of 75 percent Toys-R-Us stock and 25 percent Yahoo! stock has a beta of 1.914:

$$\beta_p = (0.75)(1.367) + (0.25)(3.555)$$
$$= 1.02525 + 0.88875$$
$$= 1.914$$

Beta examples

Company	Stock's beta, β
Allstate Corp.	0.274
General Electric	1.101
General Motors	1.173
Microsoft	1.723
Southtrust Corp.	0.419
Yahoo!	3.555

Source: Yahoo! Finance

What happens when the assumptions are not true? What most researchers have found is that even if the assumptions are violated, the general conclusions of the CAPM hold: a stock's return is best explained by the return on the market, with some adjustment for the beta of the stock.

There is debate whether the CAPM adequately describes security returns. However, there is no other generally accepted model that better describes security returns. Recent evidence questions the validity of CAPM, though there is no alternative model that is more widely accepted than the CAPM.

C. The Arbitrage Pricing Theory

The **arbitrage pricing theory** (APT) was developed by Roll and Ross as an alternative to the CAPM. CAPM has been criticized for making assumptions that are unrealistic and for not adequately explaining observed pricing anomalies.

APT has fewer assumptions than the CAPM. These assumptions are the following:

- Investors prefer more wealth to less wealth.
- Assumes capital markets are perfect.
- Investors have homogeneous expectations.
- Assumes that investors can arbitrage any pricing discrepancies.

Like the CAPM, the APT assumes that investors diversify, so the only risk that is "priced" (that is, compensated for in terms of higher returns) is non-diversifiable risk. A difference between CAPM and APT, is that in the APT the systematic risk can be measured by more than just one factor.

The APT provides that returns are generated from one of more factors, where these factors are surprises, or unanticipated changes in economic conditions. The theory does not specify how many factors drive returns, nor does it address what these factors are.

Expected returns are represented as k factors:

$$E(R_i)=\lambda_0 + \beta_{\iota 1}\lambda_1 + \beta_{\iota 2}\lambda_2 + ... + \beta_{\iota\kappa}\lambda_\kappa + \varepsilon_\iota$$

where

λ_j is the j^{th} factor

β_{ij} is the sensitivity of asset i's return to the j^{th} factor.

The β_j relate the sensitivity of asset' return to the risk factor. The λ_j is the risk premium associated with the j^{th} risk factor.

Suppose that an analyst uses the necessary statistical techniques to estimate the λ_j and β_{ij}. And suppose that β_1 = 0.3 and λ_1 = 1.5. This means that the portfolio has an exposure to the first factor of 0.3 and that the price of risk of the factor is 1.5%. This means that the first factor contributes 0.3 x 1.5 percent = 0.45 percent to the expected return of the portfolio. If portfolio manager alters the portfolio to increase the exposure from the first risk factor, the portfolio's expected return increases (as indicated by the positive value of λ_1); the portfolio manager accomplishes this by shifting investments such that the exposure to the first factor is more than 0.3.

If there is one factor – which is permissible within the APT – the CAPM may be viewed as a special case of APT (i.e., one-factor APT). The evidence regarding support for the theory is mixed. Some find that there are economic factors that drive returns, whereas others argue that the theory cannot be tested.

The macroeconomic factors found most often in research are the following:[7]

1. Unanticipated inflation
2. Unanticipated changes in industrial production
3. Unanticipated changes in the yield spread
4. Unanticipated changes in the term structure of interest rates

The reason that these factors are all unanticipated is that's the surprises that drive returns. Prices of assets already reflect the known information about the economy. It is when there is something unanticipated that causes investors to revalue assets, producing a return.

[7] These factors are observed by many researchers, including Nai-Fu Chen, Richard Roll and Stephen Ross, "Economic Forces and the Stock Market: Testing the APT and Alternative Asset Pricing Theories," *Journal of Business*, Vol. 59, No. 3 (July 1986) pp. 383-403.

Now that we know what it is, what can we do with it? The theory uses economic reasoning to describe how investors value securities. Broader than the CAPM and less restrictive in assumptions than the CAPM, the APT relates stock returns to economic factors in a very general sense.

So what good is it? Well, once we have an idea the risk exposures of a portfolio and the securities in the portfolio, we can then manage the portfolios to better manage these risks.
In this module, we've looked at the two prominent theories of asset pricing. We know that there is a risk-return tradeoff and that investors will seek out the best investment opportunities, holding well-diversified portfolios. But how do investors manage risk? What risk are they trying to manage?

In the CAPM, there is one risk that is important in asset pricing: market risk. In this model, the market factor is the one and only one factor that drives returns. This makes things very simple. If we know how a stock's returns vary according to the market's return, we can gauge its risk. And if we have an estimate of the market's return and we understand a stock's market risk, we can estimate a stock's return or assess whether a stock is under, over, or correctly priced.

But the CAPM has lots of unrealistic assumptions and the simplification of the investing world into one factor may not be appropriate. The APT is an alternative theory to the CAPM, relying less on assumptions and allowing more factors to influence returns.

Both models are depictions of how investors view risk and return. Both models rely to some extent on assumptions. And both models are supported and contradicted by empirical evidence.

So what's a portfolio manager to do? Understand the risk-return tradeoff, understand that non-diversifiable risk is what is important in determining asset prices, and understand than one or more economic factors influence the returns on assets.

2. Learning outcomes

LO7-1 Describe the capital asset pricing model;
LO7-2 Discuss the role of and relevant risk in the capital asset pricing model;
LO7-3 List and briefly discuss the assumptions of the capital asset pricing model;
LO7-4 Calculate the expected return on a security, given the risk-free rate of interest, the market risk premium, and beta.
LO7-5 Explain how arbitrage is used to explain security returns in the arbitrage pricing theory;
LO7-6 List and briefly discuss the assumptions of the arbitrage pricing theory; and
LO7-7 Distinguish between the CAPM and APT.

3. Module Tasks

A. Required readings

- Chapter 9, "Capital Market Theory," *Investments: Analysis and Management*, by Charles P. Jones, 9th edition.
- "Hedge funds", by Pamela Peterson Drake.

B. Other material

- Risk, return, and diversification, background reading from FIN3403, prepared by Pamela Peterson Drake

C. Optional readings

- Diversification and the CAPM, a PowerPoint lecture from the University of Iowa.
- The CAPM and APT, a lecture from MIT.

D. Practice problems sets

- Textbook author's practice questions, with solutions.
- Module 7 StudyMate Activity

E. Module quiz

- Available at the course Blackboard site. See the Course Schedule for the dates of the quiz availability.

F. Project progress

At this point in the semester, you should have completed the data gathering for Part C of the project.

4. What's next?

In Module 6 and Module 7, we establish the foundation for the valuation of assets. In particular, we focus on the calculation of returns and the determination and calculation of portfolio risk. In Module 8, we look at the valuation of stocks, focusing primarily on the valuation of common stocks. In Module 8, we will apply the most commonly used models in stock valuation, including the dividend valuation model and the two-stage dividend discount model. In Module 9, we will look at how we value bonds, including the role of interest rate risk and duration in valuation. In our final module, Module 10, we will focus on the use and valuation of derivatives, most notably options and futures.