



# Module 6

## Portfolio risk and return

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### 1. Overview

Security analysts and portfolio managers are concerned about an investment's return, its risk, and whether it is priced correctly by the market. If markets are efficient, the price reflects available information quickly.

A basic tenet of valuation is that the greater the investment's risk, the greater the return needed to compensate investors for that risk. But the question that arises is: What risk is rewarded by the market? Portfolio theory addresses how risk is affected when a portfolio consists of more than one investment.

#### A. Efficient Markets

An **efficient capital market** is a market in which asset prices adjust rapidly to new information. Though sometimes the price may under-adjust or over-adjust, the degree of bias is not predictable. An efficient capital market is also defined by some as a market in which all relevant information is impounded in an asset's price. This latter definition describes an **informationally efficient market**, which has the following characteristics:

- a large number of profit-maximizing market,
- these participants analyze and value securities, and
- news that may affect an asset's value is random.

#### i) The random walk

In an efficient market, stock prices are not predictable – they don't follow any particular pattern and hence there is no way to gauge the future path of prices by looking at past prices. This is because stock prices follow a random walk. A **random walk** is a time series in which the value of the series in one period is equal to the value of the series in another period, plus some random error:

$$x_t = x_{t-1} + e_t$$

where ...	which means...
$E(e_t) = 0$	The expected error is zero
$E(e_t^2) = \sigma^2$	The variance of the error is constant
$E(e_i, j) = 0$ if $i \neq j$	The correlation between the error terms of two different time periods is zero.

The implication of a random walk is that the best forecast of the  $x_t$  is  $x_{t-1}$ . If asset prices follow a random walk, then the best forecast of the value of an asset in a given period is the value of the asset in the previous period. Extending this to market trading strategies, this implies that the best forecast for tomorrow's price is today's price. In other words, it is not possible to design a trading system based on current information and consistently earn abnormal returns. An

**abnormal return** is a return on an investment in excess of that associated with the level of risk of the investment. It is the difference between the predicted return and the actual return.

In calculating abnormal returns, we must consider the amount of risk associated with the asset's value and, of course, any transactions costs. The predicted return is often estimated using the market model, which adjusts the expected return for the market's return in that period and considers the stock's market risk

## ii) Forms of the efficient market

When we refer to efficient markets, we are really referring to a set of definitions of market efficiency. We classify efficient markets according to the type of information that we believe is compounded in the price of assets: **weak form**, **semi-strong form**, and **strong form**.

EFFICIENT MARKETS AND THE IMPLICATIONS		
Type	What it means	What it implies
Weak form	Prices reflect all <u>security market information</u> .	An investor cannot trade on the basis of past stock prices and volume information and earn abnormal returns.
Semi-strong form	Prices reflect all <u>publicly available information</u> .	An investor cannot trade on the basis of publicly-available information and earn abnormal returns.
Strong form	Prices reflect both <u>private and public information</u> .	An investor cannot trade on basis of both publicly-available and private information and earn abnormal returns.

Researchers have examined stock prices for various markets to test whether or not the market is efficient.

## iii) Evidence on market efficiency

The forms of an efficient market differ according to what information we assume is already impounded in the current stock price. Testing the different forms, therefore, requires evaluating what information is contained in stock prices and what information is not.

Tests of the weak form of market efficiency involve looking at the predictability of prices based on past prices. If a trading rule could be devised to consistently earn abnormal returns, this would be evidence contrary to the weak form of efficiency. The tests of the weak form require using statistical tests of autocorrelation or runs tests. For example, if we want to test whether the prices of stocks are influenced by the phases of the moon, we would compare the returns on stocks in the different phases over time and test whether there is a difference in these prices according to the moon phase. If there is such a difference, this suggests a market efficiency and, hence, an opportunity to profit from the observed pattern of prices.

Generally, researchers have found that securities markets in the U.S. are weak-form efficient. Therefore, there is no benefit to be gained from using **technical analysis**, which relies on the use of patterns in prices. However, there are some studies that show that there exist some calendar-based anomalies that researcher are still puzzling over. An **anomaly** is a pricing situation in which an investor can earn an abnormal profit by trading in a certain manner. These possible anomalies include the:

- January effect

- Weekend effect
- Turn-of-the-year effect
- Holiday effect
- Intra-day effect
- Month-of-the-year
- Day-of-the-week

Though researchers have attempted to explain the existence of these calendar-based anomalies, they may simply be artifacts of the specific time period that was studied and not truly evidence against a weak-form efficient market.

The evidence regarding the semi-strong form is mixed, though most of the evidence suggests that prices of securities react quickly and efficiently to new information. Still, there is some evidence that raises doubts about whether prices fully reflect all available public information. Tests of semi-strong form require examining whether or not abnormal returns can be earned if an investor trades using publicly-available information after the information is released.<sup>1</sup> A test of the semi-strong form of market efficiency requires great care in adjusting for the effects of the market and for risk.

Researchers use a set of procedures commonly referred to as an **event study** to analyze prices. An event study requires estimating abnormal returns associated with an informational event. The event study generally follows the following steps:

- STEP 1: For a sample of securities, the researcher identifies the trading day on which an announcement is made. The announcement of interest may be an announcement such as a stock split, a merger, or a change in a law.
- STEP 2: The researcher collects stock returns for the days preceding, including and following the event.
- STEP 3: The researcher analyzes the stock's typical relation with that of the market in general. Usually, an extensive period such as sixty-months is used to estimate a stock's typical relation to the market.
- STEP 4: The researcher focuses on the announcement day and the succeeding trading days and measures abnormal returns.
- STEP 5: The researcher performs statistical tests on the abnormal returns to assess whether these returns are different from zero.

There are a number of studies that examine whether an **earnings surprise** is reflected quickly into stock prices. An earnings surprise is an announcement of earnings in which these earnings differ from what investors were expecting. While we expect the stock's price to increase for positive surprises and decrease for negative surprises, we expect this effect to be sudden and prices reflect the extent of the surprise very quickly. However, some studies find that there may still be opportunities to profit by trading in surprise securities after the announcement is made.

Some evidence suggests that company-specific factors can be used to predict stock market performance. For example, in a series of studies, Eugene Fama and Kenneth French document that the book-to-market value of equity ratio is related to security prices such that there is possible profitable trading opportunities from trading using this ratio to form your buy-sell

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<sup>1</sup> We would expect the stock prices to react to the information. So, for example, if a company's earnings were better than expected, we would expect the company's stock price to increase at this news.

strategy.<sup>2</sup> There is also evidence that suggests that the size of the firm is related to security prices.<sup>3</sup> However, the debate regarding whether these are truly pricing anomalies or whether they are statistical artifacts continues.

Tests of the strong form address the question: Does trading on private information lead to abnormal profits? Researchers have examined this form by focusing on the trading, for example, of:

- Corporate insider trading (the legal variety)
- Stock exchange specialists
- Security analysts
- Professional money managers

The evidence is mixed, but we can draw some general conclusions:

- If an investor has monopolistic access to information, that investor may be able to earn abnormal profits.
- Superior fundamental analysis cannot be used to generate consistent abnormal profits.

#### iv) Implications of efficient capital markets

We can draw the following conclusions from the wealth of evidence on efficient markets:

- It is not possible to earn abnormal profits from technical analysis.
- Making profits from fundamental analysis requires estimating values of assets that are better gauges of value than actual market prices, which is difficult to do.
- It may be possible to earn abnormal returns using private information, though it is not possible for the typical investor to do so and, in some cases, it is not legal for an investor to trade on inside information.

## B. Portfolio Theory

The theories related to risk and return deal with portfolios of assets. A **portfolio** is simply a collection of investments. An important concept is that combining assets in a portfolio can actually result in lower risk than the assets considered separately because of diversification.

### i) Diversification

**Diversification** is the reduction of risk from investing in assets whose returns are not in synch. Diversification is based on correlations: if assets' returns are not perfectly positively correlated, combining these assets in the same portfolio reduces the portfolio's risk.

The return on a portfolio is the weighted average of the individual assets' expected returns, where the weights are the proportion of the portfolio's value in the particular asset. The portfolio's risk is calculated considering:

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<sup>2</sup> See, for example, Eugene Fama and Kenneth French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, Vol. 46, (June 1992) pp. 427-466.

<sup>3</sup> For a review of a number of these studies, see G. William Schwert, "Size, Stock Returns, and Other Empirical Regularities," *Journal of Financial Economics*, Vol. 17 (June 1983) pp. 3-12.

- The weight of the asset in the portfolio.
- The standard deviation of each asset in the portfolio.
- The correlations among the assets in the portfolio.

In portfolio theory, we assume that investors are **risk averse**. In other words, we assume that investors do not like risk and therefore demand more expected return if they take on more risk.

## ii) Measuring risk

So how do we measure risk? One way to quantify risk is to calculate the **standard deviation of the probability distribution** of future outcomes. In the case of an investment, we are trying to gauge the risk associated with future returns on the investment.

The standard deviation of the probability distribution is a measure of risk. The standard deviation is measured relative to the expected value, which is a measure of central tendency for a probability distribution. For a given expected value, the greater the standard deviation of the probability distribution, the greater the dispersion and, hence, risk.

Calculating the standard deviation requires calculating the expected value of the probability distribution. The expected value is calculated as:

$$E(x) = \sum_{i=1}^N p_i x_i$$

The standard deviation,  $\sigma$ , is calculated as:

$$\sigma = \sqrt{\sum_{i=1}^N p_i (x_i - E(x))^2}$$

where

- $p_i$  is the probability of outcome  $i$ ,
- $x_i$  is the value of outcome  $i$ , and
- $N$  is the number of possible outcomes.

### WHAT TO CHOOSE?

Risk averse investors prefer more return to less, and prefer less risk to more.

Consider the following investments and the associated expected return and risk (measured by standard deviation):

Investment	Expected return	Standard deviation
A	10%	12%
B	10%	11%
C	11%	12%
D	11%	11%
E	9%	10%
F	12%	13%

If you are a risk-averse investor, which investment would you prefer of each of the following pairs:

A or B? C or D? D or E? E or F?

Some choices are clear, and some are not. Some, like the choice between D and E, depend on the investor's individual preferences for risk and return tradeoff, which we refer to as their **utility function**.

**EXAMPLE: CALCULATING THE EXPECTED VALUE**

*Problem*

Calculate the expected return and standard deviation associated with the following distribution:

Outcome	Probability	Return
1	20%	-10%
2	50%	20%
3	30%	40%

*Solution*

Outcome	$p_i$	$x_i$	$p_i x_i$
1	20%	-10%	-0.02
2	50%	20%	0.10
3	30%	40%	<u>0.12</u>
			$E(x) = 0.20$

Expected return = 20%

Outcome	$p_i$	$x_i$	$x_i E(x)$	$(x_i - E(x))^2$	$p_i (x_i - E(x))^2$
1	0.20	-0.10	-0.30	0.09	0.018
2	0.50	0.20	0.00	0.00	0.000
3	0.30	0.40	0.20	0.04	<u>0.012</u>
					$\sigma^2 = 0.030$

$$\sigma = \sqrt{0.03} = 0.17321 = 17.321\%$$

iii) Correlations and covariance

Covariance and correlation are statistical measures of the extent that two sets of data – two variables -- are related to one another. The **covariance** between two random variables is a statistical measure of the degree to which the two variables move together. The covariance captures how one variable is different from *its* mean as the other variable is different from *its* mean. A *positive* covariance indicates that the variables tend to move together; a *negative* covariance indicates that the variables tend to move in opposite directions.

The covariance is calculated as the ratio of the **covariation** to the sample size less one:

$$\text{Covariance} = \frac{\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})}{N-1}$$

- where
- N is the sample size
  - $x_i$  is the  $i^{\text{th}}$  observation on variable X,
  - $\bar{x}$  is the mean of the variable x observations,
  - $y_i$  is the  $i^{\text{th}}$  observation on variable Y, and
  - $\bar{y}$  is the mean of the variable Y observations.

The actual value of the covariance is not meaningful because it is affected by the scale of the two variables. That is why we calculate the correlation coefficient – to make something interpretable from the covariance information.

The **correlation coefficient**,  $\rho$ , is a measure of the strength of the relationship between or among variables. For two variables, X and Y, we calculate it as:

$$\rho = \frac{\text{covariance between x and y}}{\left(\text{standard deviation of x}\right)\left(\text{standard deviation of y}\right)}$$

$$\rho = \frac{\left(\sum_{i=1}^N (x_i - \bar{x})(y_i - \bar{y})\right) / (N-1)}{\sqrt{\frac{\sum_{i=1, n}^N (x_i - \bar{x})^2}{N-1}} \sqrt{\frac{\sum_{i=1}^N (y_i - \bar{y})^2}{N-1}}}$$

Note: Correlation does not imply causation. We may say that two variables X and Y are correlated, but that does not mean that X causes Y or that Y causes X – they simply are related or associated with one another.

or, using shorthand notation,

$$\rho_{XY} = \frac{\text{COV}_{XY}}{\sigma_X \sigma_Y}$$

where  $\rho_{XY}$  is the correlation between the returns on X and Y,  $\text{cov}_{XY}$  is the covariance of the returns on asset X and Y, and  $\sigma_X$  and  $\sigma_Y$  are the standard deviations of the returns on X and Y, respectively.

In the context of asset returns, a **correlation coefficient** is a measure of the extent to which the time series of two assets' returns tend to move together. Correlation coefficients range from -1 (perfect negative correlation) to +1 (perfect positive correlation). Correlations may be positive, negative, or zero.

## CORRELATION AND STOCK RETURNS

Consider the daily stock returns for three stocks, Dell Computer, General Motors, and Kellogg, from June 21, 2004 through June 22, 2006. These companies are in different industries, but are affected by the same general economics movements. Hence, there should be some positive correlation among the returns on these stocks.

We can calculate the returns on the stocks by downloading the daily prices and the dividends paid per share. From [Yahoo! Finance](#), we download the prices and dividends, using this information to calculate the daily return on a stock.

The daily return for a stock is calculated as:

$$\text{Daily return} = \frac{\text{Price on day } t - \text{Price on day } t-1 + \text{Dividend on day } t}{\text{Price on day } t-1}$$

Using a sample of days to demonstrate, we'll use GM stock prices and dividends:

Day	Closing price	Dividends per share	Return
9-Aug-04	\$37.18		
10-Aug-04	\$37.83		0.73%
11-Aug-04	\$37.90	\$ 0.50	-2.19%
12-Aug-04	\$37.06		-1.34%
13-Aug-04	\$36.94		4.20%

If we repeat this computation for all of the trading days and for all three stocks, we can then get an idea of the relationship between the returns on these stocks.

Using Microsoft Excel<sup>®</sup>, we calculate the correlation coefficients,  $\rho$ , using function CORREL:

	Dell	GM	Kellogg
Dell	1.00000000	0.14824737	0.13284603
GM	0.14824737	1.00000000	0.15568776
Kellogg	0.13284603	0.15568776	1.00000000

The stock returns of Dell, GM and Kellogg are positively correlated with one another. If you would like to see the worksheet that generated these correlations, accompanied by the return calculations and scatterplots, [click here](#).

### iv) Measuring portfolio risk

A portfolio's return is a weighted average of the individual asset's expected returns. That's simple. But the risk of a portfolio is much more complicated. A portfolio's risk is calculated considering the relationships among the returns of the assets that make up the portfolio. The portfolio's risk,  $\sigma_p$ , is less than the weighted average of individual asset returns' standard deviations if the returns' correlation is less than 1.0.

The portfolio standard deviation for an N-asset portfolio is:

$$\sigma_p = \sqrt{\sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_i w_j \sigma_i \sigma_j r_{ij}}$$

or, using the covariance of i's and j's returns instead of  $\sigma_i \sigma_j r_{ij}$ ,

$$\sigma_p = \sqrt{\sum_{i=1}^N w_i^2 \sigma_i^2 + \sum_{i=1}^N \sum_{j=1, j \neq i}^N w_i w_j \text{COV}_{ij}}$$

where

$w_i$  is the weight of the  $i^{\text{th}}$  asset in the portfolio,  
 $\sigma_i$  is the standard deviation of the  $i^{\text{th}}$  asset's returns  
 $r_{ij}$  is the correlation coefficient for returns of assets i and j  
 $\text{COV}_{ij}$  is the covariance for the returns of assets i and j.

For a two-asset portfolio, the portfolio risk calculation is much simpler. Consider the portfolio comprised of securities X and Y:<sup>4</sup>

$$\sigma_p = w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + 2w_X w_Y \sigma_X \sigma_Y r_{XY}$$

or, using the covariance of X's and Y's returns instead of  $\sigma_X \sigma_Y r_{XY}$ ,

$$\sigma_p = w_X^2 \sigma_X^2 + w_Y^2 \sigma_Y^2 + 2w_X w_Y \text{COV}_{XY}$$

You'll notice that the third term in each equation is what distinguishes the portfolio standard deviation from being a simple weighted average of the standard deviations of the individual asset's returns.

This is diversification at work. A portfolio's risk is reduced as you combine assets whose returns are not perfectly positively correlated. You'll notice when you work problems, the key driver in this calculation is the correlation. It is actually possible to add an asset to a portfolio that will increase the portfolio's return, yet reduces the portfolio's risk.

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<sup>4</sup> Where does the "2" come from? Consider the first formula. Because we are summing the third term from 1 to N (in this case 2) to consider the correlation of return of X with those of Y and the returns of Y with those of X, we have  $w_X w_Y \sigma_X \sigma_Y r_{XY} + w_Y w_X \sigma_Y \sigma_X r_{YX}$  that we simplify as  $2w_X w_Y \sigma_X \sigma_Y r_{XY}$ .

### Example: Portfolio risk

#### Problem

Consider the following investments A, B, and C that can be placed in a portfolio:

Stock	Expected return	Standard deviation
A	10%	20%
B	8%	15%
C	5%	10%

The correlations among these investments are as follows:

	A	B	C
A	1.00	0.40	0.80
B	0.40	1.00	-0.20
C	0.80	-0.20	1.00

What is the expected return and standard deviation for a portfolio comprised of:

1. 50% of Stock A and 50% of Stock B
2. 40% of Stock A, 40% of Stock B, and 20% of Stock C?

#### Solution

1. 50% of Stock A and 50% of Stock B

Expected return = **9%**

Portfolio variance =  $0.01 + .00563 + 2(0.003) = 0.02163$

Portfolio standard deviation = **14.71%**

2. 40% of Stock A, 40% of Stock B, and 20% of Stock C?

Expected return = **8.2%**

Portfolio variance =  $0.0064 + .00360 + 0.0004 + 2(0.00192 - 0.00024 + 0.00128) = 0.0163$

Portfolio standard deviation = **12.77%**

## C. Modern Portfolio Theory

Diversification is the foundation of **modern portfolio theory** (MPT). MPT is the theory of selecting the optimal combination of assets that are expected to provide the highest possible return for a given level of return (or least risk for a given level of return).]

Harry Markowitz developed a model that describes investors' choices. He makes several assumptions in his model:<sup>5</sup>

- Investors consider the probability distribution of expected returns.
- Investors seek to maximize their utility.
- Investors estimate risk on the basis of variability of expected returns.
- Investors base decisions solely on risk and return.
- Investors are risk averse. That is, for a given level of risk, investors prefer greater return; for a given level of return, investors prefer less risk.

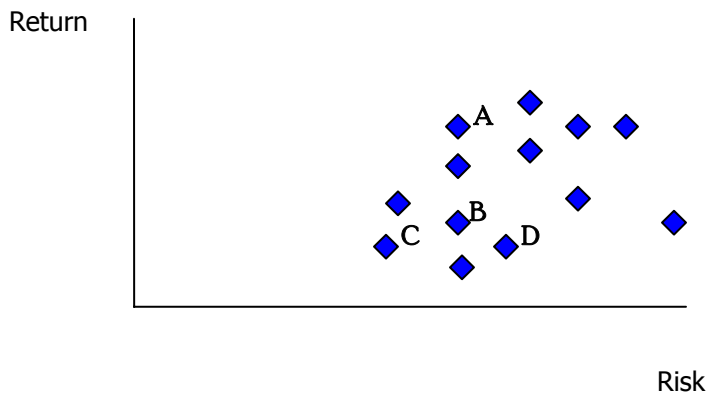
<sup>5</sup> Harry Markowitz, "Portfolio Selection," *Journal of Finance*, (March 1952) pp. 77-91.

From his model, he develops the idea that there is an optimal set of portfolios in terms of risk and return. This optimal set is referred to as the **efficient frontier**. The efficient frontier is the set of portfolios that have the greatest return for a level of risk or, equivalently, the lowest risk for a level of return. Portfolios on the efficient frontier are preferred to the "interior" portfolios.

Let's construct the efficient frontier. We will look at possible portfolios and their expected risk and expected return in the two dimensions: return (vertical axis) and risk (horizontal axis):

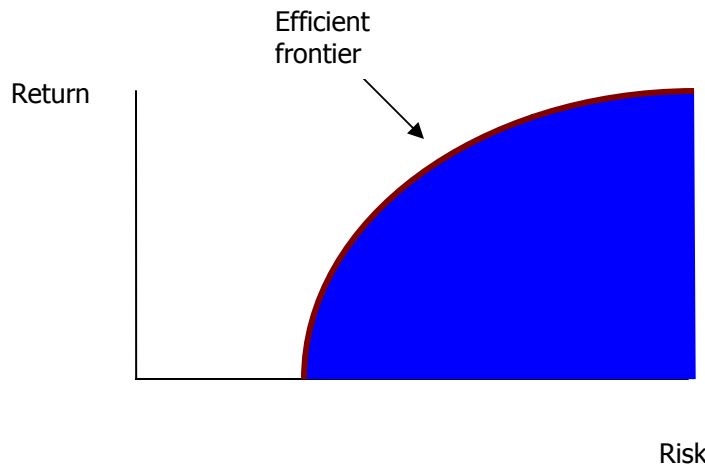


Now we calculate the expected return and risk of all possible portfolios that can be constructed using available investable assets. If we begin to plot the return-risk for each portfolio, we see the following (with representing the  $\blacklozenge$  expected return and standard deviation for a given portfolio):



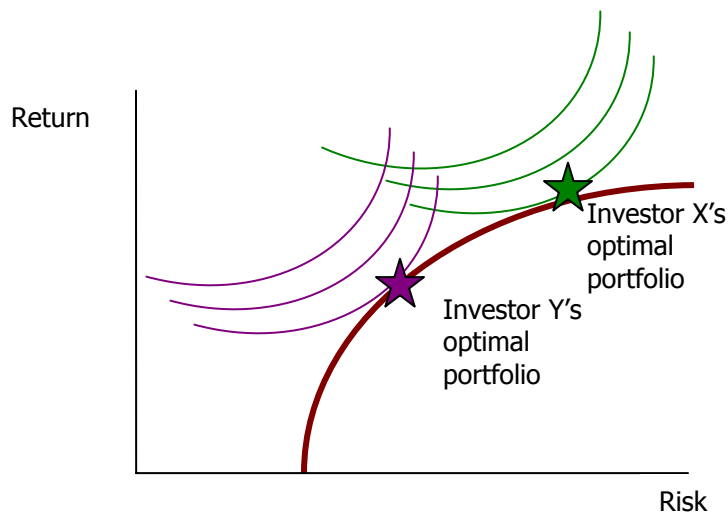
We can see in this graph that there are some portfolios that appear better than others in terms of risk and return. For example, a risk averse investor would prefer A to B (more return, same risk) and would prefer C to D (same return, lower risk).

If we keep this up, considering every possible portfolio, including all possible weights for each asset, we eventually end up with the following:



Every portfolio that lies on the efficient frontier (in this diagram, this means a portfolio that lies on the red line) is superior to the portfolios that lie interior to the frontier (in this diagram, in the blue area).

Once we have derived the efficient frontier, we can choose the portfolio on that frontier that is best for an investor. If we consider that investors have a personal tradeoff between risk and return, referred to as a **utility curve**, we can determine where the optimal portfolio lies on the efficient frontier. More risk-averse investors have steeper utility curves. The optimal portfolio is the portfolio on the efficient frontier that has the highest utility.



The optimal for portfolio for each investor is *the point of tangency between their utility curves and the efficient frontier*. The optimal portfolio represents the group of assets that maximizes the investor's utility. In other words, this selection of assets offers the greatest level of satisfaction, among all possible portfolios, for the investor considering how he or she feels about risk and return. In the diagram above, the optimal portfolios of Investors X and Y are different because of different utility functions, but they lie on the efficient frontier.

The implications of modern portfolio theory are that:

- Some portfolios are preferred to others.
- There exists an optimal portfolio for each investor.

## D. Implications

The fact that markets are efficient is often viewed as sad news among students of finance because many wish to be able to learn about securities and markets so that they could make their fortunes trading. If markets are efficient, does this mean that investment management is fruitless? No. Investment managers select investments that are appropriate for the investor's return objectives and risk objectives. Efficient markets just tell us that it is not possible to earn *abnormal* returns. Earning returns commensurate with the risk that is taken on is consistent with an efficient market.

If markets are efficient, does this mean that financial analysts do not perform a useful function? Quite the contrary. Financial analysts help investors understand the possible returns and risks associated with investments, which helps the investor choose what is appropriate for her portfolios.

## 2. Learning outcomes

LO6-1 Describe, in terms of the direction and speed of response, how stock prices react to announcements that may affect the stock's valuation.

LO6-2 Demonstrate mathematically how they interact to affect the risk of portfolios.

LO6-3 Illustrate how portfolios' risk changes as the composition of the portfolio changes.

## 3. Module Tasks

### A. Required readings

- Chapter 6, "The Returns and Risks from Investing," *Investments: Analysis and Management*, by Charles P. Jones, 9<sup>th</sup> edition.
- Chapter 7, "Portfolio Theory," *Investments: Analysis and Management*, by Charles P. Jones, 9<sup>th</sup> edition.
- Chapter 8, "Selecting the Optimal Portfolio," *Investments: Analysis and Management*, by Charles P. Jones, 9<sup>th</sup> edition.
- Chapter 6, "Technical Analysis and the Random-Walk Theory," in *A Random Walk Down Wall Street*, by Burton G. Malkiel, 1999. Available free through NetLibrary as an e-book through the Florida Atlantic University libraries.

### B. Optional readings

- Chapter 8, "A New Walking Shoe: Modern Portfolio Theory," in *A Random Walk Down Wall Street*, by Burton G. Malkiel, 1999. Available free through NetLibrary as an e-book through the Florida Atlantic University libraries. In this chapter, Malkiel discusses modern portfolio theory, risk and returns.
- [Measuring risk](#), a detailed presentation of how to calculate the expected return and standard deviation for a probability distribution, prepared by Pamela Peterson Drake

### C. Practice problems sets

- [Textbook author's practice questions, with solutions, Chapter 6](#)
- [Textbook author's practice questions, with solutions, Chapter 7](#)

- Textbook author's practice questions, with solutions, Chapter 8
- Portfolio risk and return calculations
- Module 6 StudyMate Activity

#### D. Project progress

- You should be gathering information on your company's stock price, including monthly closing prices and dividends over the past five years. You should look over the posting entitled *Estimating the market model: Step by step* and begin working through this process.

#### E. Module quiz

- Available at the [course Blackboard site](#). See the [Course Schedule](#) for the dates of the quiz availability.

### 4. What's next?

In this module, we have look at the portfolio theory and the related mathematics. This is the necessary foundation for understanding asset pricing models, which is our next topic. Following the theories of asset pricing, we look at the valuation of stocks, bonds, and derivatives.