



Module 10

Derivatives

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1. Overview

Derivative securities are investments that draw, or derive, their value from some other investment. For example, an equity option is a derivative security whose value depends on the value of the underlying stock for which there is a choice (that is, an option) to buy or sell that stock. Derivative securities have become important in both protecting the value of an investment and in speculation. These securities take on many different forms, including **forwards** (which are obligations to transact in the future), **futures** (which are standardized forward contracts), and **options** (rights to transact in the future).

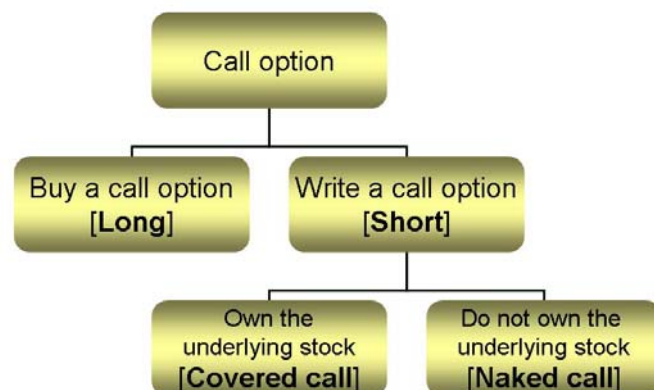
The value of a derivative security is complex because it not only depends on the value of the underlying investment, but also on the characteristics of the particular derivative contract. If the value of the derivative security in the market is not in line with the value of the underlying asset, there exists an **arbitrage opportunity**. Investors taking advantage of these arbitrage opportunities will keep the values of the derivative in line with that appropriate for the value of the underlying asset.

A. Options and option-pricing

An **option** is a contract that gives its owner the right, but not the obligation to conduct a transaction involving an underlying asset at a predetermined future date and at a predetermined price (**exercise price** or **strike price**). This differs from a futures contract, which is a *commitment* to transact in the future.

A **call option** is the right to buy the underlying asset. A **put option** is the right to sell the underlying asset. The buyer of an option is referred to as the holder, whereas the seller of the option is referred to as the writer. The buyer of the call option is referred to as having a long position on the stock, whereas the writer of a call option has a **short position**. The buyer of a put option has a position that is similar to a short position on a stock; the buyer is hoping that the price of the underlying asset will fall.

Types of call option positions



Owners of options pay for the right to buy or sell by paying a **premium**. Investors take a position in an option depending on their expectations regarding the change in the price of the underlying asset:

- The buyer of a call option expects the price of the underlying asset to increase.
- The buyer of a put option expects the price of the underlying asset to decrease.
- The writer of a call option expects the price of the underlying asset to either decrease or stay the same.
- The writer of a put option expects the price of the underlying asset to either increase or stay the same.

Prior to or at maturity – a.k.a. expiration or expiry – the investor in the option may:

1. Let the option expire worthless;
2. Exercise the option (that is, buy the asset in the case of a call option or sell the asset in the case of a put option); or
3. Sell the option to another investor.

We often refer to options' **moneyness**, which is the relation between the exercise price and the price of the underlying. For a call option,

- **Out-of-the-money** is when the asset's price is less than the exercise price.
- **At-the-money** is when the asset's price is equal to the exercise price.
- **In-the-money** is when the asset's price is greater than the exercise price.

For a put option,

- **Out-of-the-money** is when the asset's price is greater than the exercise price.
- **At-the-money** is when the asset's price is equal to the exercise price.
- **In-the-money** is when the asset's price is less than the exercise price.

The value of an option has two components:

1. the **intrinsic value**, what it is worth if exercised immediately, and
2. the **time value**, which is the present value of the expected

EXAMPLE: CALL OPTION

Consider options on Microsoft stock. On October 28, 2005, Microsoft common stock had a price of \$25.42. At that time, there were many different call and put options available with different exercise prices and expirations.

The call option with a January, 2008 expiration and a strike price of \$35.00 was selling for \$0.65. This means that:

The intrinsic value of the call option is \$0.

The time value of the call option is \$0.65 per share.

If an investor wants to buy this call option, the investor must pay $\$0.65 \times 100 = \65 for the right to buy 100 shares of Microsoft common stock at \$35 per share before the option's expiration in January 2008.

If you want to write a call option, you would receive the \$65 and be committed to sell 100 shares of the stock at \$35 per share if the option buyer chooses to exercise this option.

Let's say that the price of Microsoft stock rises to \$37 per share and the investor exercises the option. The investor's profit is:

Value of stock	\$3,700
Purchase price	3,500
Call premium	<u>65</u>
Profit	\$ 135

From the call writer's viewpoint,

Proceeds	\$3,500
Purchase price	3,700
Call premium	<u>65</u>
Profit	-\$135

If the price of Microsoft stock rises to only \$30 per share, the investor does not exercise the option. The investor's loss is \$65 and the writer's gain is \$65.

increase in the value of the option prior to expiry.

The option's **intrinsic value** is the value realized from *immediate* exercise. Let S_0 be the current stock price and let E be the exercise price. The intrinsic value for a call option is the maximum of $S_0 - E$ or 0, whereas the intrinsic value of a put option is the maximum of $E - S_0$ or 0. In other words, an option will have intrinsic value different from zero if it is in-the-money.

An option's **time value** is the amount by which the option premium exceeds the option's intrinsic value:

$$\text{Time value} = \text{options price} - \text{intrinsic value}$$

The time value depends on the amount of time remaining to expiry and the volatility of the underlying asset's value.

Options can be written on just about any asset. Consider the following options:

TYPES OF OPTIONS	
Equity options	Options on the stock of individual companies.
Stock index options	Options are traded on many different indexes (e.g. S&P 100 (OEX)). One difference between options on stocks and those on stock indexes: options on stock indexes are settled in cash.
Foreign currency options	Contracts are for the sale or purchase of a specified amount of foreign currency at a fixed exchange rate. A currency call option gives the holder the right to buy the currency at a later date at a specified exchange rate.
Options on futures	An option on a futures contract gives the holder the right to enter into a futures contract at a later date and at a predetermined price.

Trading in options takes place in over-the-counter markets and on exchanges. Organized exchanges may specialize in options on stocks and stock indexes (e.g., Chicago Board Options Exchange). The **Options Clearing Corporation (OCC)** acts as a clearinghouse for CBOE contracts, guaranteeing the writer's performance.

Example of options exchanges include:

- Chicago Board Options Exchange (CBOE)
- Philadelphia Stock Exchange
- American Stock Exchange
- Pacific Exchange
- London International Financial Futures and Options Exchange
- Tokyo Stock Exchange

Exchanges are self-regulated, but there are a number of protections built into our financial system to protect investors provided by:

- Options Clearing Corporation
- Securities and Exchange Commission
- Federal Reserve System
- Securities Investor Protection Corporation
- National Association of Securities Dealers

i) Models used to value an option

Option valuation is not as straight-forward as, say, bond valuation. The value of an option depends on several elements:

- The exercise price of the option.
- The time value of money.
- The current price of the underlying asset.
- The probability that the underlying asset's price will change in the future.
- The time remaining to expiration of the option.

There are many models that may be used to value an option. The two most widely used models are:

- a) The binomial option pricing model a.k.a. binomial lattice or binomial tree
- b) The Black-Scholes option pricing model.

These models differ in complexity and the assumptions. The models use the characteristics of the option contract, the value of the underlying asset, and assumptions about how prices of the underlying change through time to estimate the option's price.

To explore these two methods of valuing an option, please click on the buttons below.

(a) Binomial option pricing model

A **binomial option pricing model** is a method of determining the value of an option given that there are two – and only two -- possible outcomes or scenarios in any given period. We use the binomial option pricing model as an introduction to the basics of options: How an option takes on a value, given the price of the underlying asset. To use binomial option pricing, we must draw a **tree diagram** that details the paths of the underlying asset's price and the call option's value.

We construct the tree by first laying out the tree given the possible paths that the stock price may follow and then use **backward induction**^{1,2} to determine the value of a call option on that stock.

Consider an option that has an exercise price, E , of \$25 and a current price, S , of \$20. Also consider the following:

- The risk-free rate is 5%.
- The percentage change in the stock's price in an up movement is 60%.
- The percentage change in the stock's price in a down movement is 40%.
- The probability of an "up" movement, p .

¹ See, for example, Richard J. Rendleman, Jr., and Brit J. Baxter, "Two State Option Pricing," *Journal of Finance*, Vol. 34, No. 5 (December 1979) pp. 1093-1110 and John C. Cox and Mark Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, Vol. 7, No. 3 (September 1979) pp. 229-264.

² In the end, you'll be able to see that backward induction is simply common sense: Once we know what the underlying asset's prices may be, we will know what the possible values of the call option may be also.

The probability of an up movement is derived from the idea that prices move according to a **random walk**. This probability is determined using information about the risk free rate, the upward and downward movements and following formula:

$$p = (r - d) / (u - d)$$

where

r = 1 + risk-free rate of interest,

u = 1 + percentage change in the price in the up movement, and

d = 1 + percentage change in the price in the down movement.

Using the provided information on r , d , and u , the probability of an upward movement in the example is 45%:

$$p = (1.05 - 0.6) / (1.6 - 0.6) = 0.45$$

and the probability of a downward movement is the complement, or 55%:

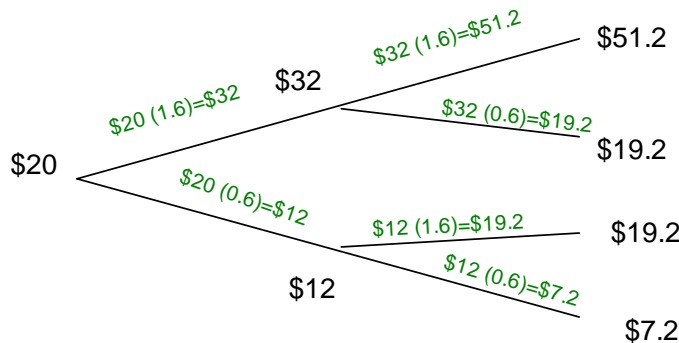
$$(1 - p) = 0.55$$

We can map out the stock price paths using the following steps:

Step 1: Calculate the stock prices at each node in the tree, using the u and d

Therefore, the stock price in the up possibility, S_u , is $\$20 (1 + 0.60) = \32 . Similarly, the stock price in the down movement is $S_d = \$20 (1 - 0.40) = \12 . The stock price in the possibility of two periods in a row of up markets, S_{uu} , is $\$20 (1 + 0.60)^2 = \51.2 ; whereas the worst case situation, in which the price goes down two periods in a row, is $\$20 (0.6)^2 = \7.2 .

Stock price tree



USING THE BINOMIAL OPTION PRICING MODEL

Step 1
Calculate the stock prices at each node in the tree, using the u and d



Step 2
Value the call option at the end of the second period stock prices



Step 3
Value the call option at the end of the first period, discounting the call option values and weighting by the probabilities associated with each branch

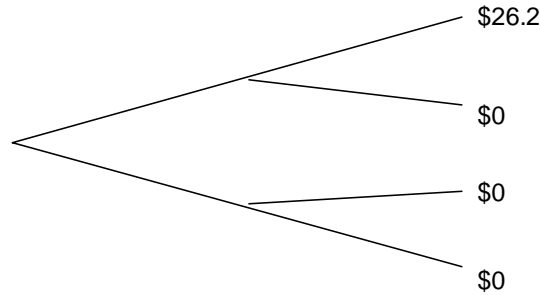


Step 4
Value the call option today by discounting the value of the call option at the end of period one and weighting by the probabilities

Step 2: Value the call option at the end of the second period stock prices

Given the stock price tree, we can value the call option on the stock, starting at the end of the second period and working backward to the present:

Call option value tree

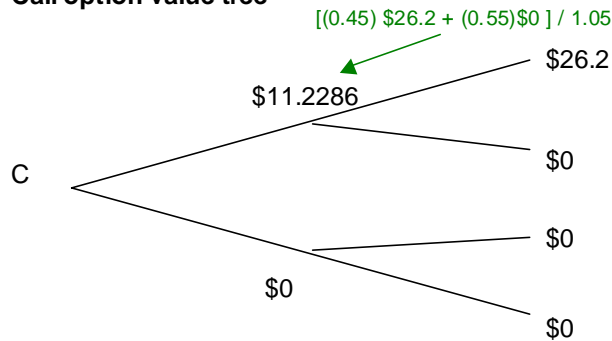


If the stock's price is \$51.2 at the end of the second period, the value of the option is $\$51.2 - 25 = \26.2 . If the stock's price is anything less than or equal to \$25 at the end of the second period, the value of the call option is \$0.

The value of the call at the end of the second period is the maximum of \$0 and $S - E$. For example, in the S_{uu} position, the value of the call is $\$51.2 - 25 = \26.2 , whereas in the S_{dd} position, the value of the call is \$0 because the stock price is not above the exercise price.

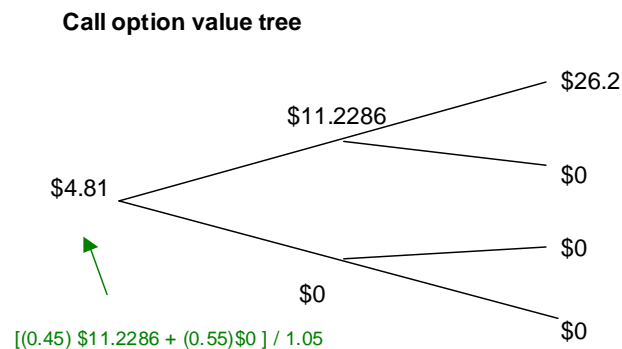
Step 3: Value the call option at the end of the first period, discounting the call option values and weighting by the probabilities associated with each branch:

Call option value tree



We discount the value of the call options, weighing the each call option's value by the probability of the movement (up or down). Because there is only one case in which the call option has an intrinsic value in this example, the calculation is rather straight-forward.

Step 4: Value the call option today by discounting the value of the call option at the end of period one and weighting by the probabilities



Therefore, the value of the call option is \$4.81.

As you can see, the value of the call option depends on the value of the underlying asset at the end of the period. In many scenarios the call option expires worthless. It is the possibility that the call option has value at expiration – in this case in the event of two up-movements (the path S_{uu}) – that gives the call option value today.

(b) Black-Scholes option pricing model

The binomial model can assist us in determining the value of an option if we assume a simple world with few, discrete time periods. But when we make things more realistic, we need a more powerful model to value an option. The **Black-Scholes model**, developed by Fischer Black and Myron Scholes, provides us with a tool to value a call option.³

The Black-Scholes model requires five inputs

1. Exercise price of the option, E
2. Price of the underlying asset, S
3. Risk-free rate of interest, r
4. Volatility of underlying asset's price, σ
5. Time to expiration, T

According to the Black-Scholes model, the value of a call option, C, is:

$$C = S N(d_1) - E e^{-rT} N(d_2)$$

where

$$d_1 = \frac{\ln(S/E) + \left(r + \frac{\sigma^2}{2} \right) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Suppose you have the following information regarding an option:

³ Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, Vol. 81, No. 2 (May-June 1973) pp. 637-654.

	Parameter	Value
Value of the underlying asset	S	\$40
Exercise price of the option	E	\$35
Time remaining to maturity	T	3
Risk free rate	r	5%
Volatility	σ	40%

What is the value of the option using the Black-Scholes model?

USING THE BLACK-SCHOLES OPTION PRICING MODEL

Step 1: Calculate the values of d_1 and d_2

$$d_1 = \frac{\ln(\$40/\$35) + \left(0.05 + \frac{0.16^2}{2}\right) \cdot 3}{0.4\sqrt{3}}$$

$$d_1 = \frac{0.1335 + 0.39}{0.6928} = 0.7557$$

$$d_2 = 0.7557 - 0.4\sqrt{3} = 0.0629$$

Step 2: Calculate the values of $N(d_1)$ and $N(d_2)$, using a normal density function table

$$N(d_1) = 0.7751 \quad N(d_2) = 0.5250$$

We can use either a **cumulative normal density function table** or the Microsoft Excel® function NORMDIST:

NORMDIST(x,mean,standard_dev,cumulative)

For example,

=NORMDIST(.7751,0,1,TRUE)

Step 3: Calculate the value of the option using the formula for the value of a call option.

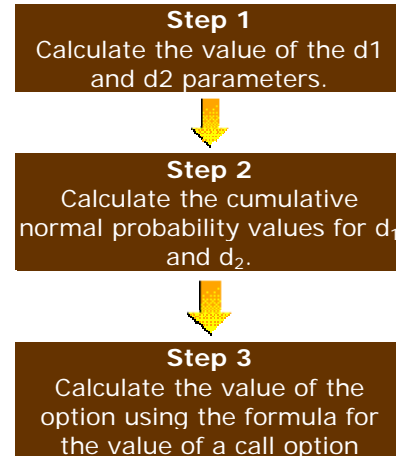
The value of the option, C, is \$15.1885:

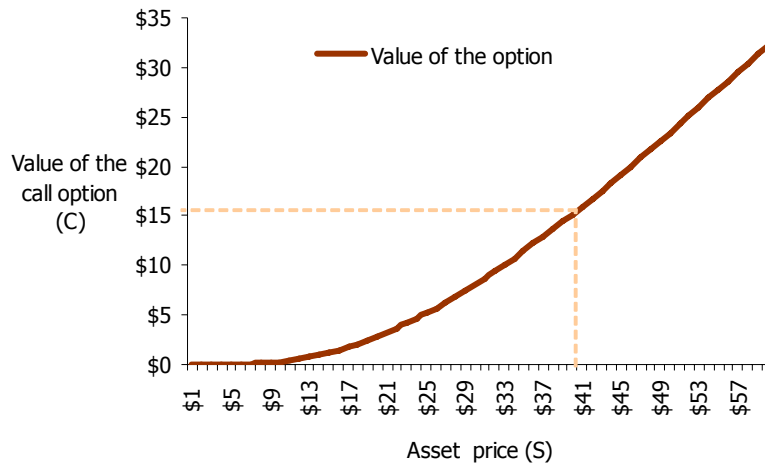
$$C = \$40 (0.7751) - \$35 e^{-0.05(3)} (0.5250)$$

$$C = \$31.0040 - \$15.8155$$

$$C = \$15.1885$$

If we look at the value of the option for different values of the underlying asset's value, we see the following:





The option has value at all asset prices (albeit a small option value) because of the potential for the underlying asset's price to increase in value in the future. The change in the value of the option for a given change in the asset's price depends on the value of the asset's price: the change in the value of the option per \$1 change in the asset's price is greater for lower prices of the asset than for larger values of the asset.

The most challenging input to the Black-Scholes model is volatility.⁴ We estimate volatility by using either historical volatility or an implied volatility. The estimate of historical volatility requires examining historical prices of the underlying asset and calculating the annualized standard deviation of these historical prices. **Implied volatility** requires looking at call prices of calls on the same underlying (but for different expiration and exercise prices) and working backward to see what standard deviation is implied given the market value of those calls.⁵



Try it

Check out the [option valuation program](#). All you need to do is input the option's:

- Exercise price
- Underlying price
- Time to maturity (in years; e.g., 6 months is .5)
- Risk-free rate of interest (in decimal form; e.g., 4% is entered in as 0.04)
- Volatility (in decimal form; e.g., 40% per year is 0.40)

The option's value is calculated for you.

		Relation to the value of a ...	
		call option	put option
Value of the underlying asset	S	+	-
Exercise price of the option	E	-	+
Time remaining to maturity	T	+	+
Risk free rate	r	+	-
Volatility	σ	+	+

Though the Black-Scholes model is quite useful in estimating the value of an option, it cannot handle dividend-paying stocks very well. However, there are models, such as that developed by Robert Merton, that incorporate cash flows, such as dividends.⁶ A

further difficulty of the Black-Scholes model is that it does not incorporate transactions costs or taxes.

⁴ For a review of the issues involved when estimating volatility, see Stewart Mayhew, "Implied Volatility," *Financial Analysts Journal*, Vol. 51, No. 4 (July-August 1995) pp. 8-20.

⁵ The calculation of implied volatility is beyond the scope of this module.

⁶ Robert C. Merton, "The Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* (Spring 1973) pp. 141-183.

In some applications, it is difficult to estimate the inputs. In the case of an option on a traded stock, there is generally no problem in estimating the five inputs. However, in other applications, such as when the underlying is not traded in an active market, it may be difficult to estimate the inputs.

ii) Option strategies

Options may be used for many purposes, including reducing risk of a particular investment or betting on future movements in the price of the underlying asset.

Key to developing an option strategy, for whatever purpose, is understanding the sensitivities of options to the characteristics of the option (e.g., time to expiration). And before we consider different strategies, we also need to understand the relation that exists between puts and calls, and how to calculate the payoffs from strategies.

The value of an option, whether we are using a binomial model or a Black-Scholes model, is affected by changes in one of the five inputs. For example, if the price of the underlying increases, the value of the option changes also. The sensitivity of an option's value to changes in the parameters is casually referred to as the "Greeks" because we represent the sensitivities with Greek letters.

The "Greeks" of Options	
Delta, Δ	Change in the price of an option for a one unit change in the price of the underlying asset
Kappa, κ a.k.a. Vega	Change in an option's price to changes in the volatility of the underlying asset
Rho, ρ	Change in an option's price to a change in the risk-free rate of interest
Gamma, Γ	Rate of change of Δ as the underlying stock price changes
Theta, θ	Change in an option's price to the passage of time

The sensitivity of the change in the price of the option to a change in the value of the underlying, **delta**, is also referred to as the **hedge ratio**. We refer to it by that name because we use the delta in designing a hedge to eliminate a risk associated with the change in the value of the underlying asset.

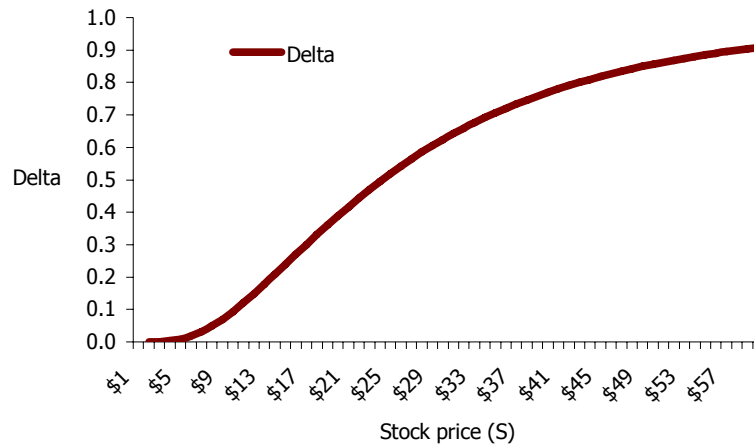
Consider the option that we valued earlier that has a current value of \$15.1885 when the stock's price is \$40. If the stock's price were to move to \$41, the value of the option, using calculation like we did before, would be \$15.9662. The delta on this option is:

$$\Delta = \frac{\$15.9662 - \$15.1885}{\$41 - 40} = 0.7777$$

As you can see by looking at the graph of the option's value relative to the stock's price, delta is different for different stock prices. For example, for this same option, if the price of the stock changes from \$20 to \$21,

$$\Delta = \frac{\$3.2349 - 2.8178}{\$21 - 20} = 0.4170$$

We can graph the delta to see how the value changes depending on the price of the underlying asset:



Delta is close to zero when the underlying asset's price is low, but approaches 1.0 as the underlying asset's price increases.

(a) Put-call parity

There is a relation between the value of a call option and the value of a put option. We refer to this relation as the **put-call parity** relationship.

This means that buying a call option (that is, "long call"), plus writing a put (that is, "short put") must be equivalent in value to investing in the stock in the cash market and borrowing to buy this investment.⁷ If this relation does not hold, there would be arbitrage opportunities between the cash (i.e., spot) market and the options market.⁸

Let P indicate the value of the put option, C the value of the call option, S the current (spot) price of the underlying asset, r the risk-free rate of interest, and T the time to maturity. The put-call-parity relation is that:

$$\begin{array}{l} \text{Buying a call option} \quad \text{Buying the stock} \\ \quad \& \quad = \quad \& \\ \text{Writing a put option} \quad \text{Borrowing the funds} \end{array}$$

or

$$C - P = S - E e^{-rT}$$

which is Algebraically equivalent to

$$P + S = C + E e^{-rT}$$

and

$$P = C - S + E e^{-rT}$$

⁷ Borrowing the present value of the underlying asset is $E/e^{rT} = E e^{-rT}$. This calculation uses continuous compounding (hence, Euler's "e").

⁸ An arbitrage opportunity exists when an investor can buy an asset in one market and sell the asset in another market simultaneously and make a profit.

This is a very convenient relationship because we can use it to solve for the value of a put option once we have calculated the value of a call option. In our example in which the stock is trading for \$40 and the exercise price is \$35, the option has a value of \$15.1885. This implies that the value of the put is:

$$P = \$15.1885 - \$40 + \$35 (e^{-0.05(3)})$$

$$P = \$15.1885 - \$40 + \$30.1248$$

$$P = \$5.3133$$

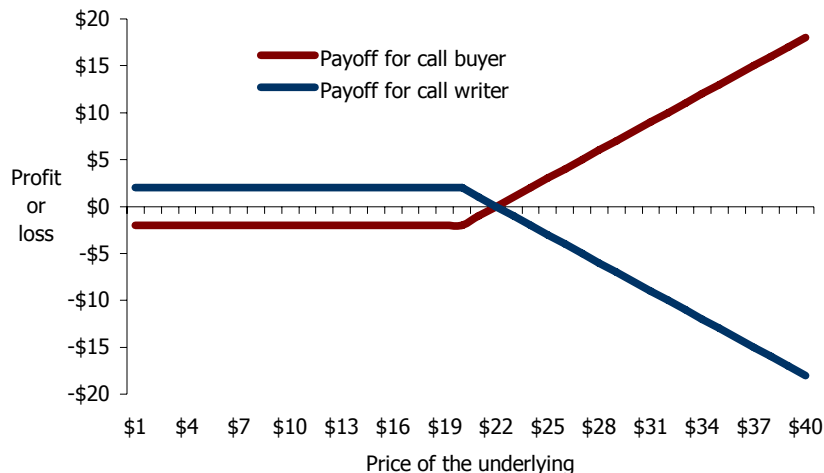
The put-call parity relation is useful in creating a **synthetic security**, which is the creation of the equivalent of one security out of combinations (buying/selling) of the others. For example, you can create a **synthetic put** by buying a call, selling the stock, and lend the proceeds of the stock sale.

(b) Option strategies

There are many option strategies that are available to investors; too many to discuss in this module. However, we'll look at some of the more common strategies in this section.

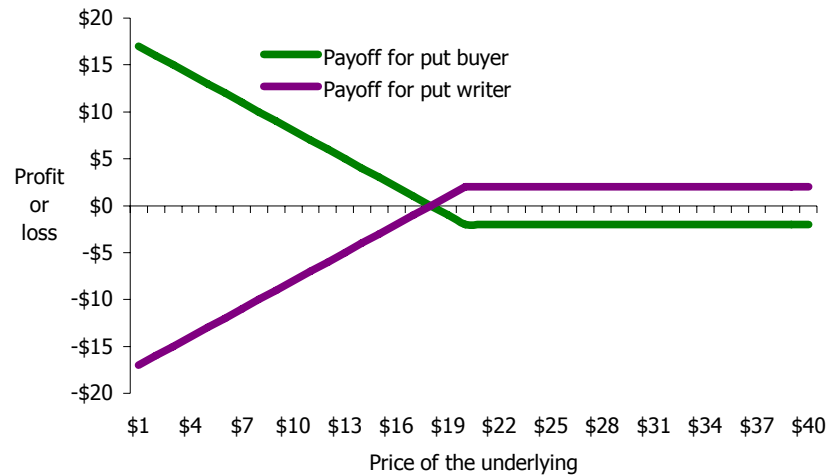
We evaluate strategies by evaluating the conditions in which they are profitable. A convenient device to evaluate strategies is to use **payoff diagrams**, which are charts that indicate the profit or loss for different values of the underlying asset.

For example, the payoff diagram for a strategy of buying a call option for \$2 that has an exercise price of \$20 is:



The maximum loss for the call buyer is \$2 and the maximum gain for the call writer is \$2. If the price of the underlying stock is \$30, the profit for the call buyer is $\$30 - \$20 - \$2 = \8 and the loss for call writer is $\$30 - \$20 + \$2 = \8 .

The payoff for a put option follows a similar symmetry with respect to the writer and the buyer of the option. Consider a put option that has a premium of \$2 and an exercise price of \$20.

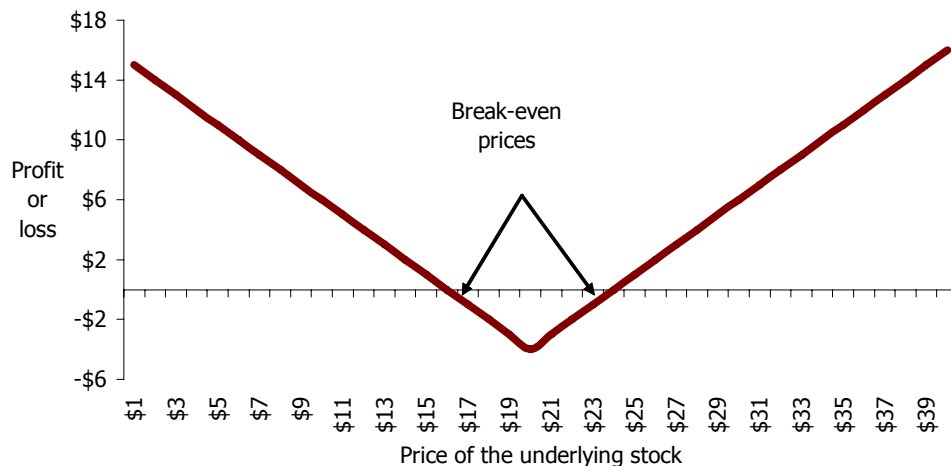


In the case of the put option, the buyer of the option makes a profit when the price of the underlying is less than the exercise price minus the premium. The writer of the put option, on the other hand, makes a profit of the put premium when the price of the underlying is at or above the exercise price of \$20.

Consider the following sampling of option strategies:

- **Long straddle.** A long straddle is one in which the investor purchases both a put and a call option for the same underlying, for which the exercise price is the same. The investor uses this strategy if he/she expects the price to change in either direction.

Consider a straddle in which the investor buys one call option with an exercise price of \$20 for \$2, and buys one put option with an exercise price of \$20 for \$2. The payoff is:

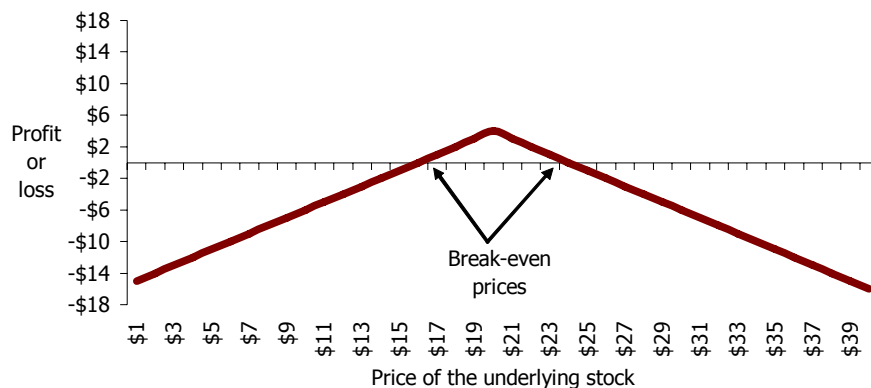


The maximum loss is \$3.50 if the price is at \$20 – the investor has paid for both options, but would not exercise them at \$20. There is profit once the price of the underlying either falls below $\$20 - 2 - 2 = \16 or above $\$20 + 2 + 2 = \24 .

To see how the payoff is determined for a strategy, we calculate the payoff for each part of the strategy and then sum the payoffs. In the straddle strategy, we can calculate the payoff for the call and the put separately, and then sum these:

Price of the underlying asset	Profit or loss on call with an exercise price of \$20 and a cost of \$2	Profit or loss on put with an exercise price of \$20 and a cost of \$2	Profit or loss on the strategy
\$11	-\$2	\$7	\$5.00
\$12	-\$2	\$6	\$4.00
\$13	-\$2	\$5	\$3.00
\$14	-\$2	\$4	\$2.00
\$15	-\$2	\$3	\$1.00
\$16	-\$2	\$2	\$0.00
\$17	-\$2	\$1	-\$1.00
\$18	-\$2	\$0	-\$2.00
\$19	-\$2	-\$1	-\$3.00
\$20	-\$2	-\$2	-\$4.00
\$21	-\$1	-\$2	-\$3.00
\$22	\$0	-\$2	-\$2.00
\$23	\$1	-\$2	-\$1.00
\$24	\$2	-\$2	\$0.00
\$25	\$3	-\$2	\$1.00
\$26	\$4	-\$2	\$2.00
\$27	\$5	-\$2	\$3.00
\$28	\$6	-\$2	\$4.00
\$29	\$7	-\$2	\$5.00

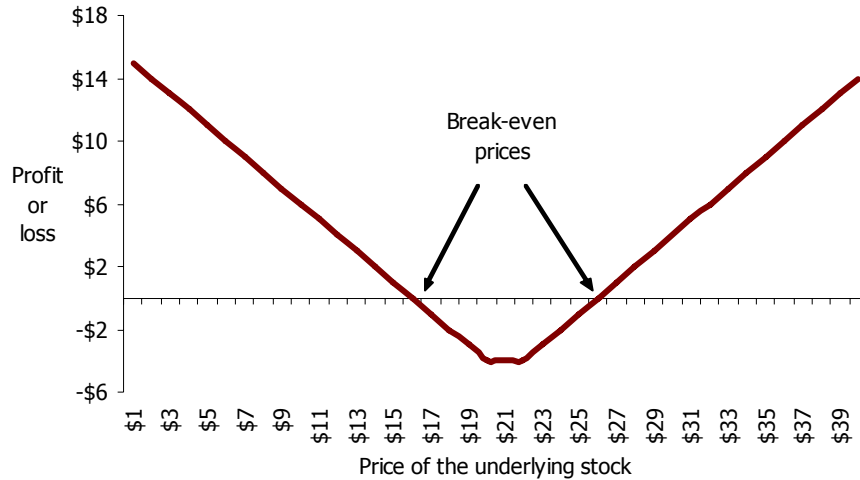
- **Short straddle.** A short straddle is one in which the investor sells both a put and a call option for the same underlying, for which the exercise price is the same. The investor uses this strategy if he/she does not expect the price to change much if at all. Consider a short straddle that involves writing a put and call, both with an exercise price of \$20 and a premium of \$2:



The investor only makes a profit if the underlying stock's price falls between \$16 and \$24.

- **Long strangle.** A long strangle is one in which the investor purchases both a put and a call option for the same underlying, for which the exercise price of the call is greater than the exercise of the put. The investor uses this strategy if he/she expects the price of the underlying to change in either direction. The investor would use a strangle instead of a straddle the greater the movement expected in the underlying asset's price.

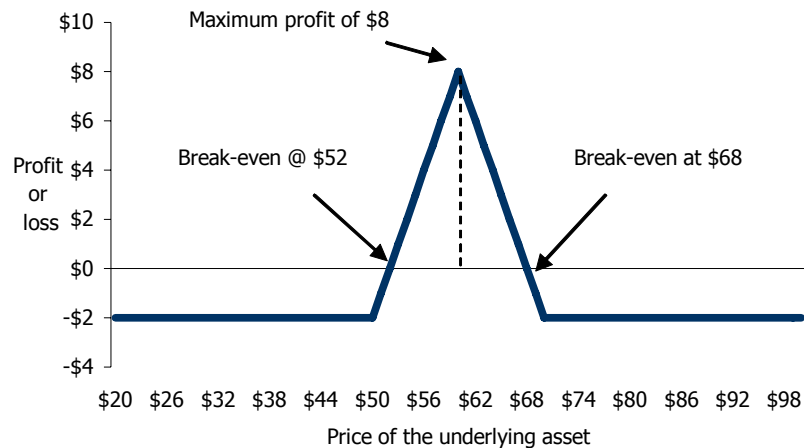
Consider a strategy of buying a call option with an exercise price of \$22 for \$2 and a put option with an exercise price of \$20 for \$2. This strategy is profitable if the underlying asset's price is less than \$16 or greater than \$26.



- **Long butterfly.** A long butterfly is a strategy that requires buying a call option with one exercise price, E_1 , buying a call option with some other exercise price, E_3 , and selling two calls with an exercise price between that of the two purchased calls, E_2 ; hence, $E_1 < E_2 < E_3$. The investor creates a butterfly spread if he/she expects the underlying asset's price not to deviate substantially from E_2 .

Consider the strategy of buying a call option with an exercise price of \$50 for \$8 and a call option with an exercise price of \$70 for \$2, as well as writing two calls with an exercise price of \$60 for \$4. This strategy produces a profit as long as the underlying asset's price is between \$52 and \$68:⁹

⁹ How do you determine the profit on the strategy? You calculate the profit of each individual strategy for each price of the underlying asset and then sum. If the underlying asset's price is \$60, the profit or loss from this strategy is $\$2 - 2 + 4 + 4 = \8 . If the underlying asset's price is \$50, the profit or loss is $-\$8 - 2 + 4 + 4 = -\2



- **Covered call.** If an investor writes a call on an asset but does not own the asset, this is referred to as a **naked call**. A covered call is the situation in which the investor owns the underlying and writes a call (that is, is short the call). The investor uses this strategy if he/she does not expect the price to move, yet wants to collect the option premium. The payoff for a covered call is similar to writing a put option.

There are other strategies that investors use beyond those that we have mentioned. When developing a strategy, it is important to understand the motivation for the strategy (e.g., reducing risk, expect that the asset's value will increase, or expecting the asset's value not to change) and the possible payoffs for different possible values of the underlying asset.

B. Forwards and Futures

Forwards and futures are types of derivative securities. A **forward contract** is a contract that gives the contract holder the right and the legal obligation to conduct a transaction for a specified quantity of an asset at a specific time in the future. A **futures contract** is a *standardized* forward contract.

Futures are traded on exchanges (e.g., Chicago Mercantile Exchange). Trading is done using the open outcry method, referred to as **pit trading**, or electronically (e.g., on GLOBEX). Traders of forwards and futures are either **hedgers**, who are reducing their risk exposure, or **speculators**, who are betting on the future price of the underlying asset.

Comparison: Forwards v. Futures

Forwards

- Private contracts
- Contracts have default risk
- Do not require cash at inception
- Not regulated

Futures

- Trade on organized exchanges
- Standardized terms
- Performance guaranteed by clearinghouse
- Require traders to post margin money
- Regulated by the government

The underlying asset in the case of futures and forwards may be commodities (such as metals, energy, or agricultural products) or financial (such as foreign currencies, debt, and equities).

A futures contract identifies:

- the underlying asset and the quality of the underlying asset, and
- the quantity of the underlying.

The exchange on which a futures contract is traded specifies:

- the minimum price fluctuation (i.e., the tick size), and
- the maximum price limits are set by the exchange.

Consider frozen orange juice futures traded on the **New York Board of Trade**. The contract for 15 thousand pounds of frozen orange juice for delivery in January 2006 was trading for \$12,050, or 80.333¢ per pound on October 28, 2005. Just two weeks earlier, prior to Hurricane Wilma, the same contract was selling for \$10,465, or 69.767¢ per pound. The futures prices are a good predictor (on a discounted basis) of the spot prices in the future. The futures prices change as expectations about the supply and demand for the underlying asset change.



FUTURES IN THE MOVIES

In the 1983 movie "Trading Places", starring Eddie Murphy, the Duke brothers attempted a scheme to use the USDA crop report information that they obtained illegally to make profits in orange juice futures contracts.

i) Hedging

Hedging is the reduction or elimination of risk associated with an asset position. A hedger has some type of risk exposure in the spot market for the asset. For example, the grapefruit grower is exposed to risk of price fluctuations during the growing season through the harvest.

The different types of positions that a hedger may take include:

- A **short hedge** is performed by selling futures.
- A **long hedge** is performed by buying futures.
- A **cross-hedge** is the use of a futures contract on an asset that is similar, but not identical from the risky asset to be hedged.

The grower of orange juice would want to *sell* futures contracts, delivering the orange juice at the expiration of the contract.¹⁰ The producer and marketer of orange juice would want to *buy* the future contract, locking in the price it pays to buy the orange juice at the expiration of the contract.

The **hedge ratio** is the number of futures contracts to hold for a given underlying cash market position:

$$\text{Hedge ratio} = \frac{\text{Futures position}}{\text{Cash market position}}$$

An investor who wishes to hedge a position will use the hedge ratio to determine how many futures contracts are needed to properly hedge the spot position.

ii) Arbitrage

The **spot price** is the price of the good today. The **futures price** is the price agreed upon for the delivery of the good at a future point in time. The **basis** is the difference between the spot or cash price of the asset and the futures price:

$$\text{Basis} = \text{spot price} - \text{futures price}$$

¹⁰ In the case of agricultural commodities, the farmer or grower will want to focus on expiration dates that are close to the harvest or slaughter dates.

The **basis** is a function of the time to maturity, costs of financing, and physical storage costs. The normal situation is that the basis is negative. That is,

$$\text{spot price} < \text{futures price}$$

and that the basis converges to zero at maturity.

There is no arbitrage opportunity if:

$$F = S (1 + c)$$

where

F is the price of the futures,
S is the price of the spot, and
c is the carrying cost.

Suppose the spot price of silver is \$3.00 per ounce. And suppose that the price of a futures contract on silver delivered in one year is \$3.50. If the cost-of-carry is 10%, is there an arbitrage opportunity? Yes.

$$\$3.50 \neq \$3.00 (1+0.10)$$

We exploit the arbitrage opportunity by buying what is relatively underpriced and selling what is relatively overpriced. How do we know which is under and over priced? Compare the two ways to invest: buy the futures contract or borrow to buy the spot for cash. Which is relatively cheaper? To see this, examine the pricing in the silver contract. The futures price is \$3.50 and borrowing to buy the silver in the cash market is \$3.30. In other words, the futures contract is over priced relative to the cash market:

$$\$3.50 > \$3.30$$

Cost in the future market > Cost in the cash market

So, we would want to sell the futures contract and buy the silver in the spot market. This is referred to as a **cash-and-carry arbitrage** because we get the cash and "carry" the underlying. A **reverse-cash-and-carry arbitrage** (sell short the underlying asset and buy the futures contract) would be used if the futures contract were underpriced relative to the cash market.

Oil Futures: What they tell us

If you bought a futures contract on Brent crude Oil, for deliver June 2007 on July 10, 2006, the cost of the contract is \$75.21 per barrel [per Yahoo! Finance].

On the same day, the spot price for Brent crude oil was \$73.99 [per WTRG Economics]. This means that the basis is -\$1.22. It is expected that the spot price of oil will converge upon \$75.21 by June 2007. In other words, investors expect the price to increase in the next year.

Example: Cash-and-Carry Arbitrage

Spot price of crude oil	= \$70	
Futures price of crude oil		= \$80
Cost of carry	= 10%	

Now

Borrow \$70.00 at 10%
Buy crude oil at \$70.00 today
Sell the futures contract (for sale of crude oil @ \$80 in the future)

Cash flow: \$70.00 - 70.00 = \$0

Later

Pay back the loan: \$70 + (10% x \$70)
Sell the crude oil at \$80

Cash flow (profit): \$80 - 70 - (10% x \$70) = \$3

2. Learning outcomes

LO10-1 Distinguish between a call option and a put option on a stock and between writing an option and buying an option and identify which options would be appropriate to use depending on an investor's expectations and objectives.

LO10-2 Explain the role of arbitrage in markets.

LO10-3 Relate the price of an option to the payoff from the option strategy.

- LO10-4 List the determinants of an option's value, relating each to an option's value, and identify these determinants for traded options.
- LO10-5 Draw and label a binomial tree for a two-period binomial option and value the option.
- LO10-6 Analyze the role of each variable in the Black-Scholes option pricing model and calculate the value of an option using this model.
- LO10-7 Draw and label a payoff diagram for strategies involving options.
- LO10-8 Analyze the role of a covered position in options and determine payoffs from uncovered and covered positions.
- LO10-9 For a given forward or futures contract, identify whether an arbitrage opportunity exists and, if so, design a strategy to exploit that opportunity and compute the arbitrage profits.

3. Module Tasks

A. Required readings

- Chapter 19, "Options," *Investments: Analysis and Management*, by Charles P. Jones, 9th edition.
- Chapter 20, "Futures," *Investments: Analysis and Management*, by Charles P. Jones, 9th edition.

B. Other material

- [Options basics](#), from the CBOE.
- [Class: Options Pricing](#), Chapters 1 through 3, by the Options Industry Council

C. Optional readings

- [Flash quiz on option payoff diagrams](#).

D. Practice problems sets

- [Textbook author's practice questions, with solutions](#).
- [Option pricing problems](#), prepared by Pamela Peterson Drake
- [StudyMate Activity](#)

E. Module quiz

- Available at the [course Blackboard site](#). See the [Course Schedule](#) for the dates of the quiz availability.

F. Project progress

Project is due by Midnight, August 4thth.

4. What's next?

The final exam.